Growth and Distribution in the Kuwaiti Economy 1960-1975: A Production Function Approach

S. Al-Qudsi*

I. Introduction

This paper presents an inquiry into the sources and nature of economic growth in Kuwait during the period 1960-1975. In the course of the inquiry, we shall deal with questions such as:

(1) What are the main sources of growth in the Kuwaiti economy for the fifteen-year period?

(2) Were forces such as technical progress and factor substitution significant growth determinants acting in concert with growth of capital and labor?

(3) Was technical progress neutral in the economy or was it biased towards uneven factor savings?

(4) How were the fruits of growth distributed among the factors of production?

We carry out our inquiry using an aggregate production function, with the hope that this approach would fill a gap that has existed in the quantitative macroeconomics of the Kuwaiti economy and shed some light on the technological aspects that have a bearing on the economy's growth performance as well as the distribution of income between capital and labor.

---

* Lecturer at the Economics Department, Kuwait University and Economics Consultant to Kuwait Institute for Scientific Research.

To be able to do so, given the binding constraint of scare data, certain assumptions have to be employed, upon which "the generation" of certain data was made. Assumption-making should not shake reliability in the estimates: for as M. Nerlove puts it, "Assumptions, indeed, are a substitute for facts." (1)

To estimate capital stock data we used the official estimates of the economy-wide capital output ratio. (2) We used the 1960 ratio as a benchmark, and then to complete the series we used the following difference equation:

$$K_t = K_{t-1} + I_g - \delta K_t \qquad (1.1)$$

where K_t + capital stock in year t,
K_{t-1} + capital stock in year t-1,
I_g + gross capital formation,
δ + official depreciation rate.

Official depreciation and GDP data are used, while interpolation and extrapolation procedures were employed to generate estimates of annual labor input. These are given in table (4.1) below.

II. Algebraic Form of the Production Function

The choice of a particular algebraic form is assoicated with the question of substitution between different inputs. It is a question concerning the curvature of the isoquants. There are several functional forms which may be potentially relevant for the analysis of the production conditions in Kuwait:

(i) Fixed coefficient (Leontief) production function.

(ii) Cobb-Douglas production function (C.D.)

(iii) Constant elasticity production function (C.E.S.)

(iv) The Translog production function (T.P.F.), and

(v) The Generalized linear production function (G.L.P.F.).

Each of the forms (i-iii) makes an assumption about the elasticity of factor substitution and, hence, implies a particular form of the factor demand equation. The Leontief and the Cobb-Douglas production functions assume substitution elasticities of zero and one, respectively. The CES function requires, in a three input case, that the marginal rate of technical substitution between any two inputs$(x_i, x_j)$to be independent level of a third input(X_K). This implies that inputs x_i andx_j are functionally weakly separable from . The CES production thus implies constant elasticity of substitution over time.

More realistically, however, the elasticity of substitution should not be constrained to take any a priori value whether zero (as in the Leontief function) or one (as in the Cobb-Douglas function) or any other constant (as in the CES function). Recognition of the severe limitation of the conventional functional forms (Cobb-Douglas, CES and the fixed proportions) has motivated substantial research to find more general forms. Both the Translog and the Generalized production functions (iv-v) above impose no separability restructions **a priori** or, equivalently, no **a priori** restrictions on the Allen elasticities of substitution between pairs of factors.

Christensen, Jorgenson and Lau (3) proposed the Transcendental Logarithmic Function (Translog). The function expresses the logarithm of output as a function if inputs in logarithms. In a two-factor case, the function takes the form:

$$\ln Y = a + \alpha \ln L + \beta \ln K + \gamma \ln L \ln K + \delta (\ln L)^2 + \varepsilon (\ln K)^2 \tag{2.1}$$

This function, which is quadratic in the logarithms of the variables, reduces to the Cobb-Douglas case if the parametersγ, δ, and ε all vanish; it also provides a second-order approximation to the CES form. (4). The function allows a greater variety of substitution than production functions (i-iii).

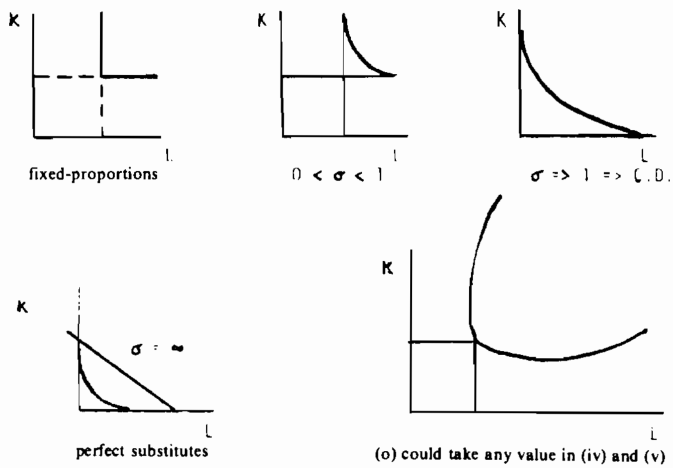
Diewert proposed the generalized (linear) production function (v).

The function is a quadratic form in an arbitrary number of inputs:

$$Y = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i^{1/2} x_j^{1/2} \tag{2.2}$$

where the parametersa_{ij} are such that $a_{ij} = a_{ji}$ and $a_{ij} \geqslant 0$ for i,j = 1, 2k ..., n. It reduces to the Leontief production as a special case. As Diewert puts it, "We may use ... the generalized linear production functions as a functional form for a production function which will attain any given set of elasticities at a predetermined set of inputs x and input prices P." (5) Like the Translog function, the generalized linear function can be viewed as a second-order (local) approximation to an any arbitrary production function. (6)

The following diagram represents the different forms of unit isoquants and elasticities of substitution implied by the above functional forms. (7)



As the diagram for the unit isoquant of the generalized linear production function indicates, elasticity of substitution could take any value. (Negative values are meaningless in a world of only two inputs, but are feasible when output is a function of more than two inputs. Some of the inputs in that case might be complements and, hence, their partial substitution elasticity has a negative value which "depends on, and varies with, the grouping of factors employed." (8)

III. Kuwait's Growth: A Production Function Approach

Which functional form of the production function is the most appropriate one is an empirical questions and could be settled by testing different functional forms on the same set of input-output data. (9) In our case, we are interested in deriving the annual distributive shares of inputs and the non-constant (over time) substitution parameter that influences them. The GLPF provides answers to these two questions and we choose to apply it to Kuwait's input-ouput data. However, the generalized linear production function is constrained by the constant returns to scale assumption, since it is homogeneous of degree one in the factor inputs. Thus, although the GLDF is one step ahead of the other functional forms by allowing for yearly variations in the substitution parameter, it is constrained to cases that exhibit constant returns to scale. Because of this last functional property, a useful and meaningful fitting of the GLPF to any industry or economy should be preceded by statistical tests of the returns to scale parameters. Only if the hypothesis of constant returns to scale is verified by these tests, can one proceed to use the GLPF. For Kuwait we do not have any reason to assume an **a priori,** constant returns to scale economy-wide production function. Therefore, we carried out some tests from which an inference about the returns to scale was made.

The test used satisfied our condition for using the GLPF. (10) It provided us with a good reason to believe that returns to scale in the Kuwaiti economy are probably constant. While the GLPF has obvious advantages over the forms (i-iii), it does not satisfy the global requirements of a well-behaved neoclassical function. These requirements are: (i) the marginal products of all factors are non-negative, which implies that production takes place in the economic region of the production (stage II), (ii) the convexity of the production function's isoquants, implying diminishing marginal rate of technical substitution and that (iii) the function must be twice differentiable so that the Hessian of the function is symmetric.

The GLPF does not satisfy these restrictions globally. It has to be checked whether the fitted GLPF satisfies these conditions at each observation.

In a two-input (capital and labor) one output (GDP) case, the function takes the following form:

$$Y = \beta_0 K + 2\beta_1 \sqrt{KL} + \beta_2 L . \quad (3.1)$$

From (3.1), the marginal productivity capital will be positive if:

(i) $\beta_0 \cdot \beta_1 > 0$

or (ii) $\beta_0 > 0, \ \beta_1 < 0$.

But $\beta_0 > \beta_1 \dfrac{\sqrt{L}}{\sqrt{K}}$

or

(iii) $\beta_0 < 0, \ \beta_1 > 0$

and $\beta_1 \dfrac{\sqrt{L}}{\sqrt{K}} > \beta_0 .$

From (4.4), the MPL will be positive if:

$$\text{(i)} \quad \beta_1 ,\ \beta_2 > 0$$

or

$$\text{(ii)} \quad \beta_1 < 0,\ \beta_2 > 0 \text{ and } \beta_1 \frac{\sqrt{K}}{\sqrt{L}} < \beta_2$$

or

$$\text{(iii)} \quad \beta_1 > 0,\ \beta_2 < 0 \text{ and } \beta_1 \frac{\sqrt{K}}{\sqrt{L}} > \beta_2 .$$

Table (3.1) summarizes the estimates obtained using the GLPF above.

Table 3.1

Generalized Linear Production Function
The Whole Economy, Kuwait 1960-1975

	Coefficient of			
Capital	$2\sqrt{KL}$	Labor	R^2	R^2
-022	42.3	-1332	.982	.98
(.190	(19)	(1876)		

Standard errors in brackets.
DW = 2.02
Number of observations = 16
Number of explanatory variables = 3.

While the coefficients of both capital and labor are insignificantly negative, their interaction term is significantly positive, and the marginal products of both factors are all postive.

Based on these estimates, the output elasticity with respect to capital over the whole period averaged 63 percent. With respect to labor output, elasticity averaged about 37 percent. Increasing capital stock by 10 percent would lead to a 6.3 percent increase in output. An increase in the stock of labor of 10 percent would generate a 3.7 percent increase in output.

From the estimated GLPF coefficients, we can analyze the relative sources of growth in the Kuwaiti economy during the studied period. The sources of growth could be estimated using the growth equation

$$\dot{Y} = (B_0 + B_1\sqrt{\frac{L}{K}})(\frac{K}{Y})\dot{K} + (B_1\sqrt{\frac{K}{L}} + B_2)(\frac{L}{Y})\dot{L} \qquad (3.2)$$

or $\dot{Y} = \alpha\dot{K} + \delta\dot{L}$

Where $\dot{Y}, \dot{K}, \dot{L}$ are the growth rates of output, capital and labor respectively, α and δ are the elasticity coefficients, i.e., $\alpha = (B_0 + B_1\sqrt{\frac{L}{K}})\frac{K}{Y}$ and $\delta = (B_1\sqrt{\frac{K}{L}} + B_2)\frac{L}{Y}$. The two terms on the right-hand side of (3.2) are the causation factors of output growth. (11) Using this approach, capital stock is responsible for about 63 percent of the economy's growth rate, labor is responsible for about 17 percent and the unexplained part (i.e., the catch-all residual) is a measure of our ignorance. (12) From the estimated coefficients we also derive estimates of the substitution parameter.

Elasticity of substitution is measured on the isoquant, which we can get by keeping output fixed. The isoquant is convex from below if:

$$\begin{vmatrix} \frac{\partial^2 Y}{\partial K^2} & \frac{\partial^2 Y}{\partial K \partial L} & \frac{\partial Y}{\partial K} \\ \frac{\partial^2 Y}{\partial L \partial K} & \frac{\partial^2 Y}{\partial L^2} & \frac{\partial Y}{\partial L} \\ \frac{\partial Y}{\partial K} & \frac{\partial Y}{\partial L} & 0 \end{vmatrix} > 0$$

Using our functional form, we have for the first year (1960):

ب

$$\begin{vmatrix} -\left(\frac{\beta_1}{2K}\sqrt{\frac{L}{K}}\right) & \left(\frac{\beta_1}{2\sqrt{KL}}\right) & \left(\beta_0+\beta_1\sqrt{\frac{L}{K}}\right) \\ \frac{\beta_1}{2\sqrt{KL}} & -\left(\frac{\beta_1}{2L}\sqrt{\frac{K}{L}}\right) & \beta_1\sqrt{\frac{K}{L}}+\beta_2 \\ \left(\beta_0+\beta_1\sqrt{\frac{L}{K}}\right) & \left(\beta_1\sqrt{\frac{K}{L}}+\beta_2\right) & 0 \end{vmatrix} \quad 0 <$$

$$= \begin{vmatrix} -.00081 & .00147 & .70 \\ .00147 & -10.66 & 1217 \\ .70 & 1217 & 0 \end{vmatrix} = 1207 \quad 0$$

Checking for local restrictions, we found that at each year the estimated function satisfies the bordered Hessian requirement. Following R.G.D. Allen (13), we compute elasticity using the definition

$$\delta_{K,L} = \frac{\partial Y}{\partial K} \cdot \frac{\partial Y}{\partial L} \Big/ Y \cdot \frac{\partial^2 Y}{\partial K \cdot \partial L}$$

Table (3.2) shows the estimated substitution coefficients over the period 1960-75.

The elasticity of substitution for th whole economy is smaller than one. For distributional purposes, this indicates that the factor whose supply is increasing at a faster rate (capital) than the other factor (labor) would experience a reduction in its distributive share. Thus, while the share of capital started off at 67 percent, it gradually declined to about 55 percent by the end of the period.

Yet official data derived from the country's national accounts show that the labor's share, besides being small, has actually been declining. Our estimates agree with the historical data in showing that labor's share is small, but disconfirm its historical tendency to decline. Our functional form (3.1) is static. Introducing a time-element to capture the effect of technical change might explain the divergence between the estimated pattern and the one observed in historical data: If technical progress is capital-using in the Hicks sense, then capital's share might increase over time, despite the low value of the substitution parameter. This can be seen by looking at the definition of the Hicks technical change (14)

$$\text{Hicks} \longleftrightarrow \left\{\begin{array}{l}\text{labor saving}\\ \text{neutral}\\ \text{labor using}\end{array}\right. \longleftrightarrow \left.\frac{\partial\left(\frac{F_K \cdot K}{F_L \cdot L}\right)}{\partial t}\right|_{\bar{K}/Y} \gtreqless 0$$

Table 3.2
Estimates of Yearly Substitution Coefficient
and Distributive Shares 1960-75

Year	K,L	Capital's Share	Labor's share
1960	.67	.69	.31
1961	.64	.655	.335
1962	.81	.666	.334
1963	.64	.654	.346
1964	.65	.658	.342
1965	.66	.655	.345
1966	.73	.653	.347
1967	.67	.65	.35
1968	.71	.634	.366
1969	.70	.63	.37
1970	.68	.62	.38
1971	.75	.617	.383
1972	.80	.617	.383
1973	.80	.59	.41
1974	.81	.56	.44
1975	.81	.55	.45

Capital's share $= [\beta_0 + \beta_1 \sqrt{\frac{L}{K}}\,] \frac{K}{Y}$

and Labor's share $= (\beta_1 \sqrt{\frac{K}{L}} + \beta_2) \frac{L}{Y}$

For the sake of detecting the potential impact of technological improvement on the augmentation of the two factors, we introduced an exponential time variable in (3.1) above as follows:

$$Y = [\beta_0 K + 2\beta_1 \sqrt{KL} + \beta_2 L] e^{\lambda_{ij} t}, \quad \lambda_{ij} = 0 \ldots .1 \quad (3.2)$$

where λ_{ij} refers to the superimposed exponential rate of technical change on the factors of production. Various values of λ_{ij} were introduced each time and the function was re-established using OLS. Our objective was to minimize the residual sum square of errors. Surprisingly enough, any factor-augmentation (i.e., capital augmented vs. labor-augmented technology or vice versa) led to a deterioration in the explanatory power of the re-estimated function. Only when the rate of technical progress was set at ten percent for both factors (i.e., Hicks' neutral technical progress) did the explanatory power of the re-estimated equation (3.2) equal the explanatory power of equation (3.1). Table (3.3) summarizes the results obtained when a Hicks' neutral technical progress was added to the right-hand side of equation (3.1)

Estimates of elasticity of substitution, output elasticities (the distributive shares) did not change when this variable was added compared to estimates obtained using (3.2) above.

Table 3.3

Generalized Linear Production Function With Technical Progress, The Whole Economy 1960-1975

Capital K	Coefficient of Capital Labor $2\sqrt{KL}$	Labor L	R^2	R^2
-.019	41.22	-1299	.985	.983
(.184)	(19.2)	(1853)		

Standard errors in brackets.
D.W. = 2.02
Number of observations = 16
Number of explanatory = 4

IV. Conclusions

The following tentative conclusions emerge from the above estimation procedure:

First, that output is more elastic with respect to changes in the amount of capital applied than with respect to changes in the amount of labor.

Second, that for the economy of Kuwait as a whole, it could be said that the production process is characterized by constant returns to scale. Over the whole period, capital appears to explain about 63 percent of the economy's growth performance, while labor and technical progress explain 17 percent and 20 percent respectively.

Third, that the elasticity of substitution between capital and labor is well below unity. From year to year, it appears to oscillate, but its oscillation range is small (between .64 and .81).

Fourth, that the technical change appears to have been of the Hicks-neutral type. This conclusion is based on many experiments with various possible paths of technical progress (i.e., exponential, constant, and arithmetic progression). The exponential path appears to fit the data best. Hicks-Neutrality leaves the ratio of the marginal productivities of the two factors unchanged and, as a result, both substitution elasticities and distributive shares do not undergo changes here vis-a-vis their values when a static function is estimated. In the GLPF a 10 percent labor augmentation and a similar capital augmentation level yield "best" result. "Best" is judged by the explanatory power of the model as well as its conformity to reality (i.e., both Hicks labor and capital saving technical progress gave distributive shares which are different from the historical data derived from Kuwait's national accounts). (15)

Fifth, the GLPF yielded distributive shares that are not constant over the period 1960-75. Labor's share is small, but tends to rise with time. This conclusion appears to be an exception to Kuznets generalized hypothesis that "The share of compensation of employees in total income tends to be higher in countries with high income per capita, lower in the less developed countries." (16) Kuwait certainly has a high per capita income, yet, judged by the structure of its economy, she belongs to the LDC's. Kuwait then, is a high per capita income, less developed country with a relatively low labor's share.

Sixth, that perhaps the most disturbing conclusion is the low labor's share vis-a-vis capital's share. (17) Over the period 1960-75, the GLPF yielded a labor share that averaged about 37 percent. Such a low share poses important questions. Given the fact that non-Kuwaitis make up about 70 percent of the total labor force, and given the presence of institutional factors that greatly restrict non-Kuwaiti's participation in property-owning and property income-generating, how much of the 37 percent labor share is the "reward" that goes to non-Kuwaitis? In other words, how much is the share of non-Kuwaiti labor in the aggregate wage bill (i.e., in the 37% of GDP)? Is the labor income distributed between the two groups equitably? If there are high inequalities between and within the two groups of workers, what are their causes? These questions, we feel, require further analysis to answer satisfactorily..

Table 4.1.

Data Used to Estimate (CD) and (GLPF)

Year	GDP	Capital	Labor *	2 KL
1960	384	434	119.529	14404
1961	479	486	131.264	15974
1962	653	545	144.163	17727
1963	679	612	156.419	19568
1964	740	680	170.000	21503
1965	749	749	184.107	23485
1966	854	854	194.105	25750
1967	872	975	214.924	38951
1968	951	1087	215.339	30598
1969	987	1186	226.537	32782
1970	962	1307	242.162	35581
1971	1347	1625	253.838	40619
1972	1562	1986	266.136	45980
1973	2112	2725	279.087	55154
1974	2456	4550	292.729	72990
1975	3279	6583	305.000	89617

* In thousands of workers. Estimated using the ASA's data, employing the equation $L(t)=L(0)e^{rt}$ where r is the observed rate of growth of labor force between two census periods.

See **Annual Statistical Abstract** 1977 Table 71 and **A.S.A.** 1976, Table 54.

The capital-output ratio for 1960 was equal to about 1.13.

FOOTNOTES

1. Nerlove, M., *Estimation and Identification of the Cobb-Douglas Functions,* Rand McNally & Co., Chicago, 1965, p. 2.

2. United Nations Economic and Social Office in Beirut, "Plan Formulation and Development Perspectives in Kuwait", *Studies on Selected Development Problems in Various Countries in the Middle East,* 1968, Table 44, 1973.

3. Christensen, L., Jorgenson, D. and Lau., "Transcendental Logarithmic Production Frontiers", *Review of Economics and Statistics,* 44. 1973.

4. In general, this function is quite flexible in approximating arbitrary production technologies in terms of substitution possibilities. It provides a local approximation to any production frontier. See Intriligator, M., *Econometric Models, Techniques & Applications,* Prentice-Hall, Inc., 1978, p. 280.

5. Diewert, W.E., "An Application of the Shepard Duality Theorem: A Generalized Leontief Production Function", *JPE,* V.70, May/June, 1971.

6. See Varian, H.R., *Microeconomic Analysis,* W.W. Norton & Co., 1978, pp. 126-128.

7. See Sato, K., *Production Functions and Aggregation,* North-Holland/American Elsevier, 1975, p. xxxi.

8. Allen, R.G.D., *Mathematical Analysis for Economists,* McMillan and Co., Ltd., 1969, p. 504.

9. Christensen, Jorgenson and Lau made an empirical study to test which production functional form is appropriate for the two-input, two-output case using aggregate U.S. data, 1929-69. They found that the use of CES is useful in representing production with two inputs and one output. The extension of this approach to two inputs and two outputs severely contradicts historical evidence. Christensen, L., Jorgenson, D. and Lau, L., "Conjugate Duality and the Transcendental Logarithmic Function", *Econometrica,* 39, 1971, pp. 255-6.

10. The test we used was simply to apply the unrestricted Cobb-Douglas function to Kuwait's input-output data. The function we used was

where A is a shift factor;
K = capital, L = Labor, and Y = GDP.
α, β are the returns to scale parameters.

The logarithmic transformation of this function was estimated. The results that we obtained are summarized below:

"Unrestricted" Cobb-Douglas Production Function, 1960-1975

Elasticity of output to:

Capital	Labor	Rate of. Technical Progress	R^2	R^2
.60	.375	3.4%	.98	.97
(.085)	(.023)	(.004)		

Standard errors in

Standard errors in brackets.
Durbin-Watson Statistics 1.74
Number of observations = 16.
Number of explanatory variables = 3.

Thus, both capital and labor are significant (the former at 95% or more while the latter at 90% or less). Technical progress is significant at 99% confidence level.

We then used the t-statistic to test the null hypothesis that the economy's returns to scale are constant:

$$t = \frac{(\alpha + \beta) - 1}{\sqrt{Var\,\alpha + Var\,\beta + 2Cov(\alpha, \beta)}} = .24 < t\,5\%\,(13\ d.f.)$$

which led us to preserve judgment on the null hypothesis that (a + B = 1).

11. See Humphries, J., "Causes of Growth", *Econ. Development and Cultural Change*, V. 24, No. 2, January, 1976.

12. These results are comparable to the ones obtained when we experimented with a Cobb-Douglas production function (i.e., labor is responsible for 18% of the growth rate, capital 62% and (neutral) technical change for 20;). See footnote (10) above.

13. Allen, R.G.D., *op. cit.,*p. 343.

14. See Sato, K., **Ibid**. An interesting discussion of this issue is given by Johnson, H., *Theory of Income Distribution,* Gray-Mills Publishing Co., 1973, pp. 46-52.

15. See El-Sheikh, R., *Kuwaiti Economic Growth of the Oil State Problems Policies, Kuwait Univ.* 1972/73 p. 144.

16. Kuznets, S., Quantitive Aspects of the Economic Growth of Nations IV: Division of National Income by Factor Shares", *Econ. Dev. and Cult. Change.*

17. To guard against the possiblity that the low labor's share is due to omitting returns to land (i.e., the government's rent from its property rights over the country's oil fields), we tested the GLPF using three inputs: labor, capital and annual extracted oil. The estimated function became non-linear in distributive shares. In other words, Euler's theorem ceased to apply to the estimated coefficients. Perhaps our weak date base is a reason for this result. The other might be that GLPF is a poor approximation to production conditions in Kuwait when more than two inputs are used.

«النمو والتوز يع في الكو يت: تحليل باستخدام دوا ل الانتاج»

يعالج هذا البحث موضوع نمو الاقتصاد الكو يتي وكيفية توز يع ثمار الانتاج من عـوامله خلال الفترة ١٩٦٠ ـ ١٩٧٥، مستهدفاً سد بعض فراغ الا تقتصاديات الكمية الكلية لهذا البلد النفطي. و بعد اجراء دراسات قياسية متعددة يضع اختيار الباحث على الداله الخطية العامة

لتمثيل اوضاع الانتاج والتوز يع الوظيفي في الفترة المذكورة. والميزة الهامة لهذه الدالـة هى انـهـا تـعامل مرونة الاحلال بين عوامل الانتاج على أنها متغر ية يأخذ قـيماً سنـو ية مختلفة. و بعبارة اخرى فلا تفترض هذه الدالة قيمة لمرونة الاحلال على نحو مسبق. من أبرز النتائج التي خلص اليها البحث أن ما ير بو على (٦٠) بالمئة من نماء الاقتصاد الكو يتي ممكن رده الى مساهمة عنصر رأس المال في حين أن دور عنصر العمل لا يجاوز (١٧) بالمئه وان أثر التقدم التكنولوجي هو في حدود (٢٠) بالمئه. وفيما يختص بالتوز يع فتدل نتائج البحث على وجود تباين في التوز يع الوظيفي بين العمل ورأس المال: فنصيب العمل لا يجاوز ثلث الناتج في حين يستحوز رأس المال على ثلث الناتج. وأما مرونة الاحلال بين العمل ورأس المال فهى صغيرة القيمة وان كانت تتغلب تغلبا طفيفا بين سنة واخرى.