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A Proposed Model for Balancing between Conceptual and Procedural Knowledge when Teaching Mathematics

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Objectives: This study aimed to introduce a proposed teaching model for balancing between conceptual and procedural knowledge. **Method:** A descriptive analysis methodology was used through reviewing a number of studies, analysing them, and identifying their findings concerning conceptual and procedural knowledge and the balance between when teaching mathematics. **Results:** In addition, a descriptive survey methodology was used. The findings of the study revealed that teachers of maths did not pay attention to the balance between conceptual and procedural knowledge during teaching. **Conclusion:** The study proposed a teaching model, based on learning theories and scientific principles, to help teachers of maths develop balance between conceptual and procedural knowledge.

Keywords: Proposed Model, Conceptual Knowledge, Procedural Knowledge, Balance, Maths Teaching.

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Introduction

In comparison with other disciplines, mathematics is characterized by its ability to be applied in various aspects of life, the nature of knowledge construction which is based on connection, sequence, depth, balance, and accurate logical order. Modern maths; however, is not merely concerned with routine processes or separate skills. Rather, it is a coherent and accurate structure based on mathematical concepts from which maths generalizations and skills are emanated (Hamzeh, 2018). Teaching maths is based on both integration and connection between knowledge construction components, i.e., no component can be taught in isolation to form knowledge construction based on understanding rather than automatic memorization. Therefore, the main aim of teaching maths is to achieve conceptual comprehension, procedural fluency, maths competency, and ability for solving problems. These merits depend largely on developing knowledge of concepts and procedures, i.e., ‘’conceptual knowledge’’, and ‘’procedural knowledge’’, respectively. For educators, there are three types of maths knowledge which play a significant role in maths education, namely practical, procedural, and conceptual (Kacmaz & Dube, 2022). Because teaching maths takes place when it focuses on understanding concepts and knowing procedures used in a lesson (Nahdi & Jatisunda, 2020). So, the conceptual knowledge means understanding concepts, facts, principles, generalizations, and mathematical connections deeply. While, the procedural knowledge means knowledge of procedures, steps, and logarithms which can be used to achieve a certain purpose and solve a problem (Star, 2005; Rittle-Johnsonn & Schneider, 2015; Rittle-Johnson, Siegler, & Alibali, 2001). That is, maths develops learners' thinking, enabling them to acquire both conceptual and procedural knowledge (Al-Qahtani, 2016).

The effectiveness of teaching and learning mathematics is based on paying attention to the development of mathematical knowledge with learners through linking and balancing between both conceptual and procedural knowledge in order to enable a comprehension-based learning to take place, as mathematical knowledge becomes meaningful to learners, which facilitates further learning, and makes mathematics important for them (Khashan, Qendeel, Khashan, Al-Natheer & Al-Salooli, 2014).

Achieving balance between both conceptual and procedural knowledge is crucial to achieve a conceptual depth and procedural fluency. This has been confirmed by the National Council of Teacher of Mathematics (NCTM, 2000) and Common Core State Standards Initiative (CCSSI) (2010) because applying mathematics correctly is based on learners' powerful conceptual comprehension and procedural fluency. However, it is difficult to separate between conceptual and procedural knowledge or be partial to either one at the expense of the other because the relationship between them is a two-way frequency where they affect each other. This means, the development of the conceptual knowledge leads to the development of the procedural knowledge and vice-versa (Rittle-Johnson, Siegler, & Alibali, 2001; Rittle-Johnson, Schneider, 2015; Bahr & Boose, 2008). In addition, knowing concepts, principles, and mathematical relations without knowing their procedural steps or knowing procedures without understanding related concepts leads to learners' misunderstanding (Khashan, et al., 2014).

A learner can never solve a second-degree formula without understanding its related concepts and rules and the converse is true. Hence, misbalancing between both types of knowledge may occur due to the existence of a mutual integration between them. This means, it is necessary to balance between both conceptual and procedural knowledge when teaching mathematics through using models, and teaching strategies and techniques. This idea has been supported by a number of studies and societies such as (NCTM, 2014; CCSSI, 2010; Rittle-Johnson et. al., 2001; Bahr & Boose, 2008; Khashan, et al., 2014; Hurrell, 2021; Zuya, 2017).

So, this study aims to explore the situation of balance between conceptual and procedural knowledge when teaching mathematics and its consequences. This can be achieved by reviewing literature, identifying the findings of some related previous studies, and proposing a teaching model to show the teaching procedures that should be followed by teachers of mathematics to achieve balance between conceptual and procedural knowledge.

Relationship between Conceptual and Procedural Knowledge

Conceptual Knowledge

According to Rittle-Johnson and Schneider (2015) and Matthews and Rittle-Johnson (2009), the conceptual knowledge refers to a series of concepts, ideas, information required in advance to form a mental or formal image of anything through interacting with environment and others. It is considered as a nucleus for forming other mathematical concepts to address ‘’whyness’’ of things occurrences. In the same vein, Danquah (2017) defined the conceptual knowledge as part of constructing effective mathematics represented in understanding the principles, which connect between multiple and gradual knowledge. Kacmaz and Dube (2022) argued that the conceptual knowledge refers to understanding maths concepts and principles explicitly and implicitly, which can be generalized to new mathematical problems. However, such knowledge requires comprehension and explanation of maths concepts and the relationship between them. Baker (2002) argued that the conceptual knowledge of the learner has a connection with his/her ability that enables him/her to interpret phenomena of defining concepts and their relationship with solving mathematical problems, and applying signs and symbols that represent the concepts or interpret assumptions and interrelationships of these mathematical concepts.

In addition, Hakim, Alghadari & Widodo (2019) argued that learners need to understand the requirements of a mathematical problem, before planning to solve it. That is, they can connect information provided with what they are looking for; the case that allows them to solve this problem. There are many skills learners should show to solve problems such as understanding mathematical terms, connecting ideas, planning appropriate strategies, and reading and analysing graphs, algorithms, etc. These skills form the conceptual knowledge that can help learners solve problems. Moreover, the conceptual knowledge refers to both explicit and implicit understanding of mathematical ideas and what they contain of concepts, facts, principles, rules, mathematical relationships and mechanism of interrelationship between them to show their mathematical principles and the way to be used when doing procedures and solving mathematical problems.

Components of Conceptual Knowledge

The mathematical conceptual knowledge involves a number of mathematical elements and components learners must know. Many researchers in mathematics education (Hechter, 2020; Abu Odeh, 2018; Rittle-Johnson & Schneider, 2015; Zulnaidi & Zakaria, 2010; Star, 2005) argued that the conceptual knowledge includes the following:

- Understanding mathematical concepts and relationships between them.
- Understanding terms and symbols and their meanings.
- Understanding interrelationships between major and minor mathematical concepts.
- Understanding principles, generalizations, facts and rules, and their components and the relationships between them.
- Providing examples and non-examples of mathematical concepts and principles.
- Showing mathematical concepts in a variety of ways such as shapes, graphics, etc.
- Explaining the relationships between mathematical concepts and principles.
- Having a conceptual depth through perceiving concepts deeply.
- Understanding ways of employing mathematical concepts, rules and principles, practically and mathematically.

Importance of Conceptual Knowledge

According to Hurrell (2021), Hakim and Yasmadi (2021), and Rittle-Johnson, et. al. (2001), the conceptual knowledge makes learning easier and clear, which, in turn, helps learners remember, apply, and connect between new and existing knowledge. They argued that the conceptual knowledge is important for learners because it organizes their perceptions of mathematical ideas; forms primary foundations that move them to a deeper level of thinking; facilitates understanding the reasons behind both correctness or incorrectness of a mathematical step; contributes to understanding both procedures and steps used to solve mathematical problems; helps them make cognitive network connections in more than one form; makes mathematical ideas meaningful, which provides them with allies to be used in their real life situations; organizes their knowledge cumulatively; develops their skills of interpreting mathematical ideas from one form to an equivalent one; represents concepts or interprets assumptions and relationships that contain two or more concepts; uses, connects, processes and differentiates between concepts; and compares, distinguishes and integrates these concepts with rules.

So, there is a direct relationship between conceptual knowledge and mathematical achievement, i.e., the deeper the learner's conceptual knowledge is, the greater his/her mathematical achievement is (Zulnaidi & Zakaria, 2010).

Procedural Knowledge

If the conceptual knowledge is concerned with ''whyness'', i.e., why something is correct, the procedural knowledge is concerned with ''howness'', i.e., knowing the way to know the correctness of something (Rittle-Johnson & Star, 2007). In this connection, Rittle-Johnson et al. (2001) defined the procedural knowledge as familiarity with individual symbols of a mathematical structure. For Star (2005), it refers to the ability to use such symbols in solving a mathematical problem logically confirming that solving mathematical problems, in general, is a higher thinking because the procedural knowledge determines what the appropriate solution strategy is, and how it is applied in a mathematical problem. This knowledge shows flexibility in writing generalizations, and mathematical rules in a number of forms. Rittle-Johnson and Schneider (2015) considered it as a skill, which helps learners use a series of procedures to solve a problem rather than the ones that require a direct application of a rule. In this regard, they admitted that solving mathematical problems is not always procedural knowledge. For example, $x + 3 = 5$ is not procedural knowledge because it does not need many sequential steps to be solved; rather, it can be solved in one step only. Thus, the procedural knowledge is that knowledge, which manifests the learner's ability to use steps or algorithms in an organized way to solve a mathematical problem. This means, the procedural knowledge deals with specific problems that require sequential steps to be solved.

Components of Procedural Knowledge

Procedural knowledge refers to the learner's knowledge of procedures, operations, mathematical skills and steps adopted for applying them. Some educators (e.g., Hechter, 2020; Rittle-Johnson & Schneider, 2015; Al-Halisi & Al-Suloli, 2016; Zulnaidi & Zakaria, 2010; Star, 2005; Schneider & Stern, 2010) argued that the procedural knowledge includes: recognizing and understanding steps for applying mathematical operations or solving mathematical problems without addressing the solution or implementing plans to reach the goal directly because it is the knowledge used for doing a certain thing rather than implementing it; understanding mathematical algorithms and implementing their methods; understanding connections incurred between steps for solving mathematical problems; awareness of choosing appropriate methods for solving a mathematical problem or implementing a mathematical skill; knowing the purpose of doing operations or mathematical problems themselves; knowing the sequence of the procedural steps used, algorithms, and problem solving; knowing structures, which means the learner's awareness of how to construct certain sentences, draw a specific model, design a specific plan, or install a computer, i.e., awareness of construction steps and structures; understanding rules of implementing algorithms and mathematical skills; awareness of mathematical connections between skills, processes and mathematical problems and their life applications; and realizing a correct solving for a mathematical problem through explaining the reasons behind correct steps used.

Importance of Procedural Knowledge

The importance of procedural knowledge for learners is based on the fact that it can increase their awareness of the nature of a problem solving, which is not limited to one rule; rather, it can be modified based on the available data. The procedural knowledge can develop learners' prediction skills through enabling them to adapt what they understand in any new similar task. Moreover, it provides them with opportunities that help them choose appropriate solution methods, use similar methods, and increase mastery of conceptual knowledge through repetitive solutions (Rittle-Johnson & Star, 2007; Rittle-Johnson & Schnder, 2015).

Relationship between Conceptual and Procedural Knowledge

Mathematical competence depends on learners' ability to develop and connect between their conceptual and procedural knowledge. To study such a connection, it is important to evaluate each one independently, and show the different opinions about this connection. There are four different opinions and each one is supported by an evidence:

First Opinion: The findings of studies conducted by Hakim and Yasmadi (2021) and Baroody and Mix (2006) confirmed that an increase of the conceptual knowledge may lead to an increase in the procedural one. They added that without understanding concepts, learners may do procedures automatically falling short at explaining the steps used and the results found.

Second Opinion: A study conducted by (Haapasalo, Kadijevich, 2000) showed that the procedural knowledge encourages learners to indirectly explain the solutions produced, enables them to apply the conceptual knowledge, and reduces misconceptions. In the same vein, (Manondhar et al., 2022) found that the level of the conceptual knowledge increases when the procedural knowledge is repeated.

Third Opinion: A study done by (Radu, 2002) concludes that there is an independent relationship between conceptual and procedural knowledge; that is, the growth of one does not affect the other. Similarly, Baroody, et al. (2007) believed that the procedural knowledge is self-contained and does not depend on the conceptual knowledge. To prove this true, they provided an example on users who deal with smart devices, where they do not need in-depth knowledge of mathematical equations to deal with programmes practically.

Fourth Opinion: The relationship between conceptual and procedural knowledge is repetitive and bidirectional which is based on a balance between them (Rittle-Johnson, et al., 2001; Rittle-Johnson & Star, 2007; Rittle-Johnson & Schnder, 2015; Qetrani, et al. 2021; Kadijevich, 2018). That is, the relationship between conceptual and procedural knowledge is binary where an increase in one leads to an increase in the other. This was supported by Baker (2002), which showed that there is a two-way relationship between them, which appears more clearly when teaching mathematical equations (cf. equivalence principle). Put differently, improving in the conceptual knowledge helps learners produce a variety of procedural answers while the multiplicity of solutions leads to explaining concepts in various ways. In other words, each one affects, and gets affected by, the other. On this base, Kieran (2013) argued that both conceptual and procedural knowledge are intertwined, and thus, should not be separated from each other in teaching.

Importance of Balancing between Conceptual and Procedural Knowledge

The researchers believe that the fourth opinion is the logical one that should be taken into consideration when teaching mathematics, because the relationship between conceptual and procedural knowledge is integrative and based on a balance between them. That is, each one affects the other and can never be separated from it.

After reviewing the relevant literature, most studies necessitate balancing between conceptual and procedural knowledge when teaching mathematics, despite the different opinions of educators about the relationship between them, to decide which one comes first, or whether they come together. However, most of empirical and theoretical studies confirm that the frequently bidirectional relationship between them is based on a balance. In this regard, Rittle-Johnson, et al. (2001); Rittle-Johnson, Schneider (2015); Rittle-Johnson, Schneider, Star (2015) argued that the relationship between conceptual and procedural knowledge is a frequently bidirectional one. That is, each knowledge affects the other where the development of conceptual knowledge helps in the development of procedural knowledge and vice versa. The NCTM (2000) and the CCSSI (2010) recommended that teaching mathematics should develop conceptual and procedural knowledge in a balanced way to achieve procedural fluency and mathematical competence.

A study conducted by Hurrell (2021) concluded that the conceptual and procedural knowledge should be presented in a balanced way where each one supports the other, giving priority to the conceptual knowledge to be acquired by learners before the procedural one, but the opposite is risky. Another study conducted by Hussein (2022) revealed that mathematics teachers believed that both conceptual and procedural knowledge are of the same importance, so, achieving a balance between them is necessary to understand mathematics. This was confirmed by Star (2005) when necessitating a balance between conceptual and procedural knowledge rather than focusing on one at the expense of the other. However, Khalil et al. (2021) stressed that conceptual and procedural knowledge can be introduced together in a balanced way when reviewing previous experiences confirming that teaching behaviours should be correct and balanced when presenting concepts and skills of a mathematics lesson. A study was conducted by Nahdi and Jatisunda (2020) argued that learners succeed in learning mathematics when they are able to combine between conceptual and procedural knowledge well. In this regard, Puma et al. (2023) confirmed that when dealing with maths procedures, concepts and procedures should be used together. Their study revealed that the indicators of both conceptual and procedural knowledge were balanced when using the four mathematical processes, showing no priority for any of them over the other. As recommended by studies of Khashan et al. (2014), Ibrahim (2013), and Zuya (2017), there is a need to design teaching models and strategies to balance between presentations of conceptual and procedural knowledge when teaching and evaluating mathematics. Kridler (2012) recommended that mathematics teachers should develop learners' both types of conceptual and procedural knowledge in a balanced way when planning and implementing teaching, because connecting the procedural knowledge with the conceptual facilitates learning, shortens calculations, reduces errors and forgetting, leads to learning retention, and enhances mathematical efficiency. Based on the importance of balancing between conceptual and procedural knowledge, the question, which may arise is: What is the extent of balancing between conceptual and procedural knowledge in mathematics teachers' performance?

Reality of Balancing between Conceptual and Procedural Knowledge when Teaching Mathematics

To identify the reality of balancing between conceptual and procedural knowledge when teaching mathematics, some literature and related studies were reviewed. Based on the analysis and findings, it was found that mathematics teachers did not focus on achieving balance when introducing conceptual and procedural knowledge in mathematics lessons; rather, they tended to focus on the procedural knowledge. Further, there was a lack of awareness of the importance of balancing teachers' conceptual and procedural knowledge. This was confirmed by the findings of some studies (Bahr & Boose, 2008), which showed that there were contradictions in mathematics teachers' opinions and practices concerning balancing between conceptual and procedural knowledge. However, they showed some misunderstanding about the balance between them, which, in turn, grouped their opinions into two sections: each section focused on one type of knowledge. However, the findings of Al-Halisi & Al-Suloli (2016) showed that the level of teaching practices for conceptual knowledge by mathematics teachers was average while it was higher for the procedural one. Concerning the level of balancing between the two types of knowledge, it was low, showing that the teachers' focus was on the procedural knowledge during the assessment tests administered.

Another study was conducted by Ibrahim (2013) which found that mathematics teachers focused on the procedural knowledge when teaching mathematics, showing a lack of balancing between conceptual and procedural knowledge. Similarly, Khashan et al. (2014) found that mathematics teachers tended to focus on the procedural knowledge more than conceptual knowledge when teaching mathematics. In comparison, Hussein (2022) revealed that mathematics teachers tended to use traditional teaching methods that focused on procedures and neglected the conceptual knowledge. Kridler (2012) found that mathematics teachers were interested in developing the two types of conceptual and procedural knowledge, but they were introduced separately which failed to enable their learners to connect between concepts and procedures. Manandhar (2018) showed that developing knowledge in mathematics often focuses on procedural knowledge in comparison with the conceptual knowledge because mathematics teachers used teaching methods and strategies that placed weight on the procedural knowledge and reduced the conceptual one.

Based on what has been said, mathematics teachers focused on the procedural knowledge when teaching mathematics without paying attention to make a balance between the two types of knowledge. This, from our point of view, has a significant effect on learners because they are considered as machines that perform mathematical operations with no awareness of cognitive and conceptual bases. This was confirmed by Rittle-Johnson, et al. (2001) when arguing that when learners develop automatic memorization for procedures, they fail to deeply comprehend concepts and ideas that facilitate application of what they learn. They added that such a type of learning is more prone to forgetting. Moreover, when teachers pay more attention to the procedural knowledge during teaching, the level of learners gets higher in such an aspect but it may get lower in the conceptual knowledge and comprehension showing multiplication of common conceptual errors, producing, as a result, difficulties in mathematics. Star (2005) believed that mathematics teaching is based on the procedural knowledge rather than the conceptual one, which may develop mathematical skills without real understanding and knowledge. In so doing, it may interrupt learners rather than benefiting them. In support of this view, a study was conducted by Ghazali and Zakaria (2011) showing that learners got a higher level in the procedural comprehension while it was lower in the conceptual one. Similarly, Meqdadi et al. (2013) found that the overall level of learners in both conceptual and procedural knowledge was low although the latter was higher than the former. In the same vein, Cheng-Ya et al. (2013) showed that learners' level of performance in the procedural knowledge was higher than in the conceptual one. However, Al-Salooli (2013) found that the level of mathematics teachers in the conceptual knowledge was low. The findings of Zoya (2017) showed that the student teachers' performance in the conceptual knowledge in algebra was low while in the procedural one was significantly higher. Givvin, et al. (2011) argued that that learners' mathematical knowledge was largely procedural, showing ineffective mathematical thinking and creating a desire to perform incorrect or partially correct procedures. Finally, Al-Enezi (2020) found that the overall level of conceptual and procedural knowledge with learners was low.

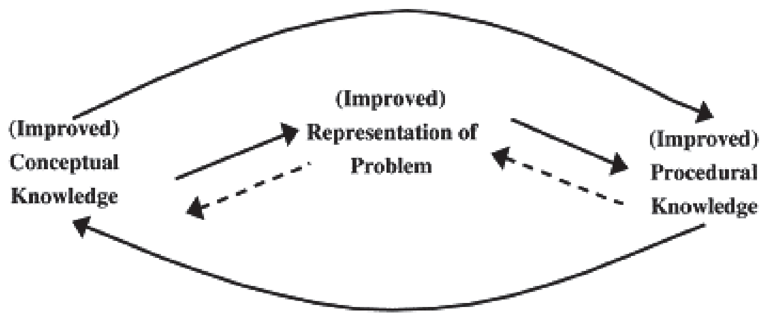
Based on the findings of the previous studies listed above, the level of balancing between conceptual and procedural knowledge when teaching mathematics was low because teachers paid more attention to the procedural knowledge than to the conceptual one. This means, mathematics teachers do not realize the importance of balancing between the two types of knowledge due to the lack of awareness of procedures that can help them balance between the two types of knowledge during planning, implementation or evaluation stages. Ignoring this aspect (balancing) renders the level of learners in both conceptual and procedural knowledge even if they show a slightly good level in the procedural knowledge. From the researchers' point of view, all these findings form a problem in mathematics teaching, which motivates them to design a teaching model that may help mathematics teachers balance between both conceptual and procedural knowledge.

Models of Balancing Conceptual and Procedural Knowledge

Efforts of some educators available on the Net on models of conceptual and procedural knowledge have been reviewed by the researchers. It has been found that some models show how conceptual and procedural knowledge are introduced and integrated with each other side by side during the teaching process. The most important models is the Frequent Model which is developed by Rittle-Jenson et al. (2001) for developing the relationship between conceptual and procedural knowledge (see Fig. 1). That is, both conceptual and procedural knowledge develop frequently to improve the representation of the problem. This model shows that the construction of conceptual and procedural knowledge is based on frequency, and the development of one leads to the development of the other. See Fig. 1 below:

Fig. 1

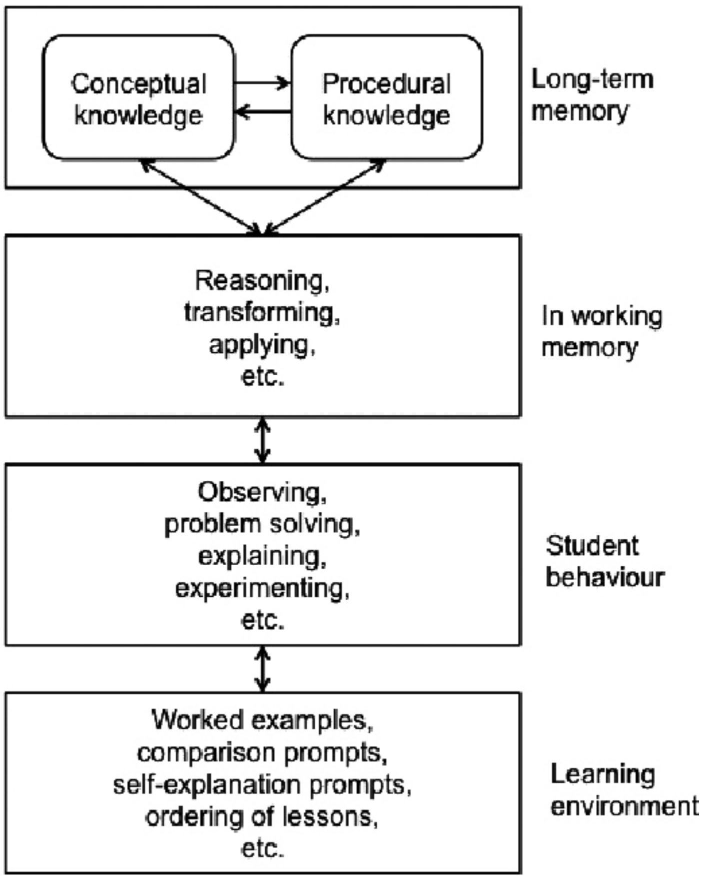
Frequent Model (Rittle-Johnson, et al., 2001)



Rittle-Johnsonn and Schneider (2015) introduced Information Processing Model for processing information related to the relationship between conceptual and procedural knowledge, which shows that both types of knowledge are stored independently in the learner's long-term memory and how it is changed by his/her experience, how he/she employs it and transfers it into application in the working memory through his/her behaviour (e.g., observation, discovery, experimentation for solving problems, etc.). See Fig. 2 below:

Fig. 2

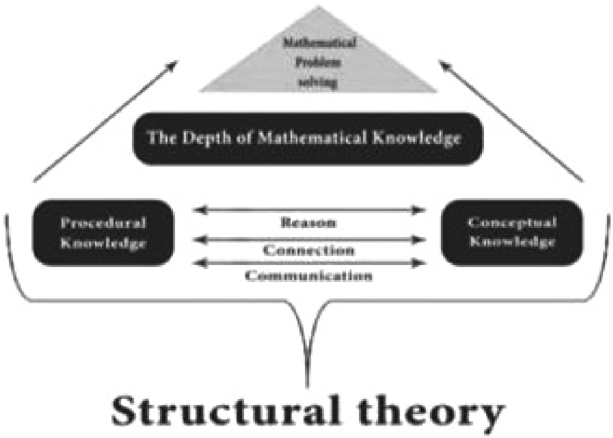
Information Processing Model (Rittle-Johnson & Schneider, 2015)



Another teaching model was developed by Khalil (2022), which introduces the mathematical conceptual and procedural knowledge in the primary education through the standards of mathematical power operations (i.e., mathematical communication, coherence, and inference) for shifting to the conceptual depth of the mathematical knowledge. See Fig. 3 below:

Fig. 3

Khalil Model



The above models are considered good unique attempts developed by researchers to show the mechanism of teaching mathematical knowledge in a way that ensures a balance between conceptual and procedural knowledge. However, there are some criticism levelled against these models. The Frequent Model, introduced by Rittle-Jenson et al. (2001), provides a general view of the direction of the bilateral and frequent relationship between the two types of knowledge only and did not provide detailed procedural steps for balancing between them. From our point of view, it is general, which may provide a mathematics teacher with a partial benefit which may not help him/her apply it in teaching. Whilst, the Information Processing Model of Rittle-Johnsonn and Schneider (2015) may help teachers motivate learners show their previously existing conceptual and procedural mathematical knowledge, use it in various mathematical situations, and solve mathematical problems. But it does not provide a procedural mechanism that can be applied to show teachers how to introduce conceptual and procedural knowledge in a balanced manner in teaching. Finally, Khalil Model (2022) is considered a general model that does not provide detailed procedural steps that show how to balance between procedural and conceptual knowledge.

So, these models are good attempts, despite their shortcomings, which form the real nucleus that motivates the researchers to propose a teaching model that may help mathematics teachers balance between conceptual and procedural mathematical knowledge in teaching. This model is mainly procedural and applicable as it involves all stages of teaching mathematics: planning, implementation, and evaluation. Applying these stages by teachers ensures achieving a balance between the two types of knowledge.

Research Questions

The Research attempts to address the following questions:

1 - What is the situation of balancing between conceptual and procedural knowledge when teaching mathematics?

2 - What is the proposed teaching model for balancing between conceptual and procedural knowledge when teaching mathematics?

Objectives of the Study

The Research aimed to:

1 - Explore the situation of balancing between conceptual and procedural knowledge when teaching mathematics.

2 - Introduce a proposed teaching model for balancing between conceptual and procedural knowledge when teaching mathematics.

Significance of the Study

The significance of the study emanates from the importance of teaching conceptual and procedural knowledge of maths and balancing between them when teaching maths. That is, the study contributes to:

1 - Showing the nature of conceptual and procedural knowledge of maths and the relationship between them better.

2 - Attracting teachers' attention to the importance of balancing between conceptual and procedural knowledge to be able to teach students in a way that achieves balancing between them.

3 - Introducing a clear and real picture of the nature of teaching conceptual and procedural knowledge and the mechanism of balancing between them to be helpful for interested people in maths education.

4 - Introducing a procedural teaching model that helps maths teachers in balancing between conceptual and procedural knowledge when teaching mathematics.

Delimitations of the Study

This study limits itself to designing a proposed teaching model, composed of three areas (planning, implementation, and evaluation), to achieve a balance between conceptual and procedural knowledge of maths to help maths teachers of basic education when teaching their students. This model was introduced to a number of maths education experts at some Yemeni universities in the academic year 2022 – 2023.

Research Methodology

To achieve the objectives of the study and address its research questions, a descriptive analysis was used. That is, a number of related previous studies on both conceptual and procedural knowledge and the balance between them were reviewed. Then, they were analysed to show the meaning of conceptual and procedural knowledge, show the nature of relationship between them, analyse the situation of achieving balance between them, and discuss their findings. In addition, a number of studies which introduced models for teaching maths knowledge and studying the allies of balancing between both conceptual and procedural knowledge, such as Rittle-Johnson et al. (2001) (see Fig. 1), Rittle Johnsonn and Schneider (2015) (see Fig. 2), and Khalil (2022) (see Fig. 3), were reviewed. All these models were analysed to come up with a more comprehensible and procedural model to achieve balance between both conceptual and procedural knowledge when teaching maths along the three teaching stages of planning, implementation, and evaluation. Then, a descriptive research methodology was used through introducing the model to a sample of 10 experts and specialists in maths education to test its importance in teaching maths, steps, and stages. Using their opinions, the model was finally designed (see Fig. 4).

The Proposed Teaching Model for Balancing between Conceptual and Procedural Knowledge when Teaching Mathematics

Due to the importance of balancing between procedural and conceptual knowledge when teaching mathematics, the researchers proposed a teaching model which consists of a set of teaching procedures to be implemented by mathematics teachers in an integrative manner at all stages of mathematics teaching to balance between procedural and conceptual knowledge. This model is composed of the following:

- **Objectives:** It is a procedural model for teaching mathematics which copes with the mathematics teaching stages (planning, implementation, and evaluation) and provides sequential procedural steps in an algorithmic style that helps mathematics teachers apply it in order to achieve a balance between procedural and conceptual knowledge.
- **Principles and Foundations:** The model has been constructed based on a number of principles and foundations: connecting previous conceptual and procedural experiences with the new one and deepening it, emphasizing coherence and balance of mathematical knowledge structure in all stages of the lesson, encouraging learners

to employ conceptual and procedural mathematical knowledge in real life situations and problem solving, balancing between teaching and evaluation of procedural and conceptual knowledge, encouraging learners to infer connections between concepts and procedures, providing information, ideas and skills in a logical sequence, showing flexibility in introducing conceptual knowledge and its skill applications or vice versa, and providing learners with activities that motivate them to discover and infer the conceptual and procedural aspects of the relationship between them.

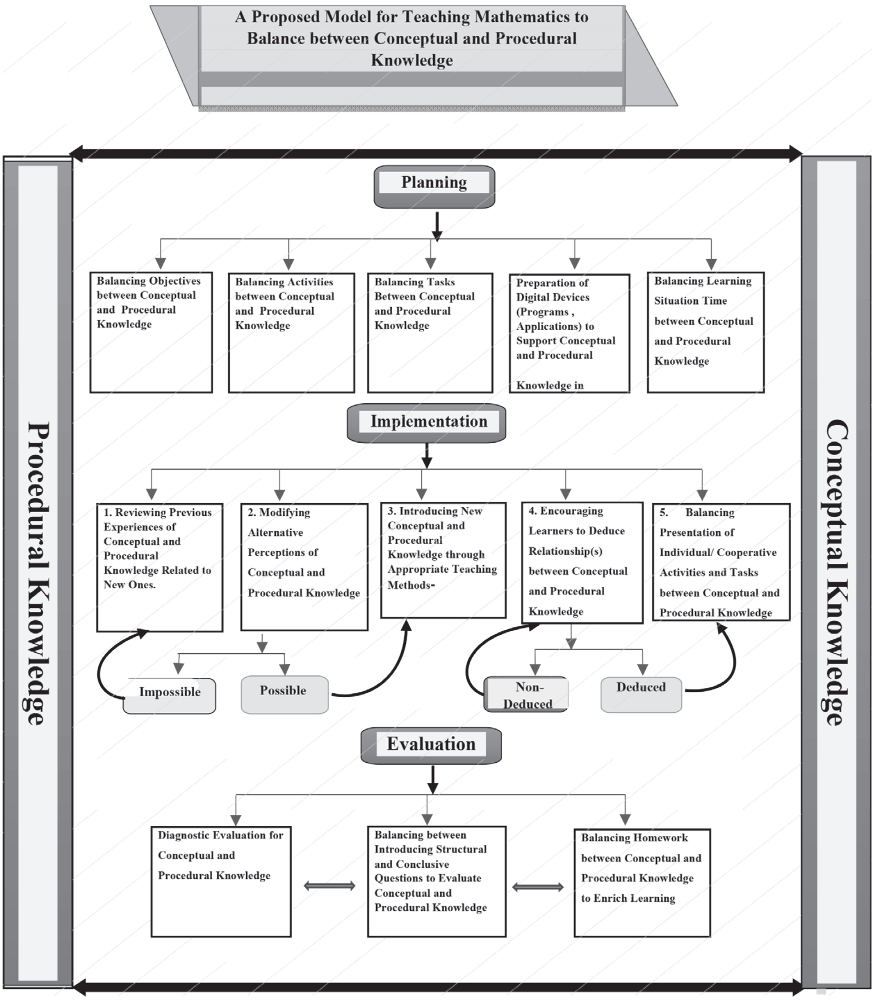
- **Philosophy:** The philosophy of this proposed model is based on the principles of learning theories and their applications in teaching and learning mathematics. The model makes use of the principles of cognitive theories that focus on knowledge and its construction mechanism, and support the mathematical knowledge structure coherence such as the theories of Osbel, Brunner and Gagnier. In addition, it makes use of cognitive and social constructivism which shows the importance of both constructive and active roles of the learner in constructing knowledge as a main concern of the educational process.

Teaching Stages of the Proposed Model

This proposed model consists of three main teaching stages, namely planning, implementation, and evaluation. In each stage, there are procedures and techniques in the form of algorithms depending on the roles of both teacher and learner. Fig. 4 summarizes these stages:

Fig. 4

A Proposed Model for Balancing between Conceptual Knowledge and Procedural Knowledge



Stage I: Planning

This stage involves procedures for teaching conceptual and procedural mathematical knowledge to achieve a balance between the two. The most important procedures a mathematics teacher should do are: designing learning objectives distributed equally between conceptual and procedural knowledge, and preparing learning aids (e.g., devices, applications, and programmers, etc.) to support conceptual and procedural knowledge in a balanced manner.

Stage II: Implementation

This stage involves procedures for teaching conceptual and procedural mathematical knowledge to achieve a balance between the two. The most important procedures a mathematics teacher should do are:

1 - Reviewing previous experiences of conceptual and procedural knowledge, connecting them with new ones through introducing questions that motivate the recall of learners' previous experiences (i.e., concepts, principles, theories, rules, skills, etc.), and then discussing their answers on this base.

2 - Providing alternative perceptions through correcting misconceptions of conceptual and procedural knowledge incurred by learners and addressing difficulties encountered. If comprehension takes place, the teacher explains the next step; otherwise, he/she goes back to the previous step.

3 - Introducing new conceptual and procedural knowledge considering flexibility and any frequent connection between the two. It is possible to start with the conceptual knowledge and connect it with its procedural answer; or, the other way round, i.e., to start with the procedural knowledge and connect it with its conceptual aspects. In this step, the teacher is required to use modern teaching strategies that cope with the nature of mathematical knowledge and learners' individual differences using a learner-centered approach. This can be achieved through using conceptual maps, exploration, constructive learning, etc. In addition, the teacher is required to use modern techniques such as projectors, interactive digital videos, mathematics digital applications, and the like in a balanced manner to display conceptual and procedural knowledge.

4 - Encouraging learners to infer connections between concepts and procedures using exploration, investigation, and induction, and reinforcing them with techniques.

5 - Providing learning activities, exercises, assignments and tasks about the conceptual and procedural aspects in a balanced manner, and encouraging learners to perform them individually or collaboratively. At this point, the teacher is required to follow-up learners' performance continuously and provide appropriate feedback and reinforcement.

Stage III: Evaluation

This stage involves procedures that examine the extent of achieving the objectives designed to evaluate the conceptual and procedural knowledge and the balance between them. This can be done through:

1 - Administrating a diagnostic assessment for learners in both conceptual knowledge and procedural knowledge to identify strengths and deficiencies, so as to reinforce the former and remedy the latter immediately through confirming the conceptual knowledge and correcting procedural steps or algorithms.

2 - Providing exercises and problems for both conceptual and procedural equally and hierarchically from simple to complex.

3 - Assigning balanced home assignments for both conceptual and procedural knowledge to support and build up learners' progress, and provide them with opportunities to develop self-learning and build balanced mathematic power.

Example of Teaching a Triangle Area Lesson Using the Proposed Model

- The teacher determines the aspects of the conceptual and procedural knowledge of a triangle area lesson in a balanced way. The aspects of the conceptual knowledge are, for example, knowing a triangle area, height, and base, based on the type of the triangle (e.g., right angle, equilateral, isosceles, etc.); the relationship between the triangle height, base and sides; and the formula of calculating a triangle area, formula components, and symbols. However, the aspects of the procedural knowledge are, for example, drawing and identifying a triangle height, measuring its length and base, explaining the steps of calculating a triangle area, showing the mathematical processes done for this purpose, etc. That is, there should be a balance between the two types of knowledge.

- The teacher states balanced objectives between conceptual and procedural knowledge of a triangle area lesson. Then, he/she uses balanced learning activities, teaching strategies, and learning techniques to achieve the objectives of the lesson.
- The teachers reviews the previous experiences, knowledge, concepts, and procedures of students to check their understanding in such aspects. Then, he/she connects them with the aspects of the conceptual and procedural knowledge of the new lesson. For example, he/she can check learners' awareness in the meaning of a triangle, its types, lengths, the relationship between them, triangle theories (cf. conceptual knowledge), etc. Furthermore, he/she can test learners' abilities in drawing all types of triangles, measuring sides and angles, etc. (cf. procedural knowledge).
- The teacher shows what is meant by a triangle area to leaners by using videos and manual concretes, and geometric boards. Then, he/she asks them about that to check their understanding. Furthermore, he/she shows learners the different types of triangles, using a board, data show, or concretes, telling them to calculate the lengths of triangles and identify the differences between them. Next, he/she explains what is meant by a triangle height, and how to draw and measure it. The same techniques can be used by the teacher to explain a triangle base (cf., a balanced explanation of both conceptual and procedural knowledge).
- The teacher writes the formula of a triangle area in words and in symbols as follows:

  In words: The area of a triangle is equal to half the product of its base and height.

  Symphonically: $A = ½ (b \times h)$
- Then, he/she shows the conceptual components of the formula and its symbols. That is, A denotes the area of a triangle, b denotes the base of a triangle, and h denotes the height of a triangle.

- Next, he/she draws a number of triangles and tells learners to draw heights and write their symbols, and identify bases and write their symbols. He/she can show learners how to calculate the area of a triangle step by step through providing a number of examples using some geometric programmes (e.g., Geo Gebra) to make them understand it very well (cf. conceptual and procedural knowledge).
- The teacher tells learners to do individual and group activities on drawing several triangles, identifying heights and bases, and measuring lengths. Then, he/she tells them to explain how they use the formula to calculate the area. Furthermore, he/she can tell them to calculate the area of each triangle, supporting them with feedback and reinforcement continuously (cf. a balanced conceptual and procedural knowledge).
- The teacher provides learners with a number of exercises and life problems in relation to the calculation of a triangle area. Then, he/she tells them to explain the steps of solving these exercises, identify the conceptual components of the problems, and then solve them. In so doing, a balanced understanding of conceptual and procedural knowledge can be reinforced.
- The teacher provides learners home assignments on the conceptual and procedural aspects of a triangle area in a balanced way.

Conclusion

Conceptual and procedural knowledge is necessary in teaching mathematics to achieve conceptual depth, mathematical efficiency, and ability to solve mathematical problems. This, in turn, requires some attention to be paid to develop conceptual and procedural knowledge, and connect them together focusing on their frequent connections to achieve a balance between them. In this study, the researchers proposed a teaching model consisting of several stages and procedural steps in a simplified algorithmic way that helps mathematics teachers achieve a balance between both conceptual and procedural knowledge when teaching mathematics.

Recommendations and Suggestions

It is recommended that maths teachers should use this teaching model to achieve balance between conceptual and procedural knowledge when teaching maths, and get trained on modern maths strategies that may help them achieve balance between conceptual and procedural knowledge. For further research, the researchers suggest conducting a study to identify the effectiveness of this proposed teaching model in achieving balance between conceptual and procedural knowledge.

Reference

Abu-Ooda, A. M. (2018). *The level of conceptual and procedural knowledge for teaching maths in the basic education with student teachers at Islamic University, Gaza*]Unpublished M.A. Thesis[. Faculty of Education, Islamic University, Gaza, Palestine.

Al-Enazi, H. (2020). The degree of showing conceptual and procedural knowledge with the 2nd intermediate students of mathematics. *Scientific Journal of Faculty of Education*, Assuit University, *36*(11), 123 – 141.

Al-Haleesi, S., & Al-Salooli, M. (2016). The situation of teaching practices for conceptual and procedural knowledge by maths teachers in intermediate education. *International Journal of Specialized Education, 5*(7), 354 – 372.

Al-Qahtani, O. (2016). The Effectiveness of the Suggested Program Based on the Constructivism Theory in Developing the Performance of Primary School Mathematics Teachers at Tabuk Region. *The Educational Journal*, Academic Publication Council- Kuwait University, *31*(121), 273- 318.

Al-Salooi, M. S. (2013). Investigating the conceptual knowledge of calculus with high school mathematics teachers. *Journal of Education Message and Psychology, 40*, 41-57.

Bahr, D., & Bossé, M. (2008). The State of Balance Between Procedural Knowledge and Conceptual Understanding in Mathematics Teacher Education. *International Journal of Mathematics Teaching and Learning, 140*, 1-28. https://www.cimt.org.uk/journal/bossebahr.pdf

Baker, W. (2002). Written meta-cognition and procedural knowledge. *Educational Studies in Mathematics, 32*, 1-36.

Baroody, A., Lai, M., & Mix, K. (2006). *The Development of Young Children's Early Number and Operation Sense and its Implications for Early Childhood Education*. https://psycnet.apa.org/record/2005-14862-011

Baroody, A.J., Feil, Y. & Johnsonn, A. (2007). An alternative reconceptualization of procedural and conceptual knowledge. *Journal for Research in Mathematics Education, 38*(2), 115-127. https://doi.org/10.2307/30034952

Cheng-Yaa, L., Jerry, B., Der-Ching, Y. & Tsai-Wei, H. (2013). Preservice Teachers Conceptual and Procedural Knowledge of Fraction Operation: *A comparative Study of the United State and Taiwan. School Science and Mathematics, 113*(1), 41-51.

Common Core State Standards Initiative. (2010). Common Core State Standards for mathematics (CCSSM). http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Danquah, P. (2017). *Conceptual and Procedural Instruction: Mathematical Teaching Approaches and Strategies in an Urban Middle School* [Unpublished PhD Theses]. University of New England. https://dune.une.edu/theses/101

Ghazali, N., & Zakaria, E. (2011). Students' procedural and conceptual understanding of mathematics. *Australian Journal of Basic and Applied Sciences, 5*(7), 684- 691.

Givvin, K.B., Stigler, J.W., & Thompson, B.J. (2011). What community college developmental mathematics students understand about mathematics, Part II: The interviews. *The MathAMATYC Educator, 2*(3), 4–18.

Haapasalo, L., & Kadijevich, D. (2000). Two types of mathematical-knowledge and their relation. *Journal for Mathematics, 21*(2), 139-157.

Hakim, L., & Yasmadi, B. (2021). Conceptual and Procedural Knowledge in Mathematics Education. *Design Engineering, 9*, 1271-1280.

Hakim, L.L., Alghadari, F., & Widodo, S.A. (2019). Virtual manipulatives media in mathematical abstraction. *In Journal of Physics: Conference Series, 1315*(1).

Hamzeh, M. (2018). The Effect of Using Stepans of Conceptual Change Model on the Development and Reser-vation of Mathematical Concepts and Acheive- ment of Tenth Grade Students in Jordan. *The Educational Journal,* Academic Publication Council- Kuwait University, *32*(128), 301- 331.

Hechter, E. (2020). The relationship between conceptual and procedural knowledge in calculus [Unpublished PhD Theses]. University of Pretoria. http://hdl.handle.net/2263/78457

Hurrell, D.P. (2021). Conceptual knowledge OR Procedural knowledge OR Conceptual knowledge AND Procedural knowledge: Why the conjunction is important for teachers. *Australian Journal of Teacher Education, 46*(2), 57- 71. http://dx.doi.org/10.14221/ajte.2021v46n2.4

Hussein, Y. (2022). Conceptual Knowledge and Its Importance in Teaching Mathematics. *Middle Eastern Journal of Research in Education and Social Sciences, 3*(1), 50-65. DOI: https://doi.org/ 10.47631/mejress.v3i1.445

Ibrahim, I. R. (2013). Teaching balance patterns between conceptual and procedural knowledge with mathematics teachers and their effect on the teaching effectiveness. *Journal of Mathematics Education, 16*(2), 132 – 171.

Kacmaz, G., & Dube, A. (2022). Examining pedagogical approaches and types of mathematics knowledge in educational games: A meta-analysis and critical review. Educational Research Review, (35), 1-15. https://doi.org/10.1016/j.edurev.2021.100428

Kadijevich, D. M. (2018). Relating Procedural and Conceal Knowledge. *The Teaching of Mathematics, XXI* (1), 15–28. http://elib.mi.sanu.ac.rs/files/journals/tm/40/tmn40p15-28.pdf

Khalil, I. (2022). A Proposed Model to Teach Mathematical Knowledge in the Primary Stage. Conference: NCTM Research Conference January, 7, 2022. DOI:10.13140/RG.2.2.22998.98883

Khalil, I., Issa, A., Al-Maleki, M. & Al-Natheer, M. (2021). The effect of a proposed teaching model on developing maths acquisition and conceptual understanding in the light of Ausubel theory with elementary 4th class pupils. *Journal of Islamic University for Educational and Psychological Studies, 29*(1), 378 – 398.

Khashan, K. H., Qendeel, R. A., Khashan, M. M., Al-Natheer, M. A., & Al-Salooli, M. S. (2014). The balance between conceptual and procedural knowledge and the factors that affect it with maths teachers in elementary education in KSA. *Journal of Educational Sciences, 26*(2), 287 – 310.

Kieran, C. (2013). *The false dichotomy in mathematics education between conceptual understanding and procedural skills: an example from algebra. In K. R. Leatham (Ed.)*, Vital directions for mathematics education research (pp. 153-171). New York, NY: Springer.

Kridler, P. (2012). *Procedural and Conceptual Knowledge: A Balanced Approach?*. Unpublished PHD Theses, George Mason University, Fairfax, VA.

Manandhar, N. (2018). *Conceptual and Procedural Knowledge Of Students In Mathematics: A Mixed Method Study* [Unpublished Master Theses]. Kathmandu University, Dhulikhel, Nepal.

Manandhar, N. K., Pant, B. P., & Dawadi, S. D. (2022). Conceptual and Procedural Knowledge of Students of Nepal in Algebra: A Mixed Method Study. *Contemporary Mathematics and Science Education, 3*(1), 1-10. https://doi.org/10.30935/conmaths/11723

Matthews, P. & Rittle-Johnsonn, B. (2009). In pursuit of knowledge: Comparing reflex plantations, concepts and procedures as pedagogical tools. *Journal of Experimental Child Psychology, 104*(1), 1-21. https://doi.org/10.1016/j.jecp.2008.08.004

Meqdadi, R. M., Malkawi, A. R., & Al-Zoghbi, A. M. (2013). Conceptual and procedural knowledge of fractions and their relationship with student teachers' anxiety of mathematics. *Journal of Educational Sciences Studies, 40*(2), 1555 – 1570.

Nahdi, S., & Jatisunda, M. (2020). Conceptual Understanding and Procedural Knowledge: A Case Study on Learning Mathematics of Fractional Material in Elementary School. *Journal of Physics: Conference Series, 1477*, 1- 5. doi:10.1088/1742-6596/1477/4/042037

National council of teacher of mathematics (2000). principles and standards for school mathematics. https://www.nctm.org/uploadedFiles/Standards_and_Positions/PSSM_Executive Summa

NCTM. (2014). *Principles to actions: Ensuring mathematical success for all. Reston: National Council of Teachers of Mathematics*, Inc.

Puma, S., Sander, E., Saumard, M., Barbet, I. & Latouche, A. (2023). Reconsidering conceptual knowledge: Heterogeneity of its components. *Journal of Experimental Child Psychology, 227*, 1-18. https://doi.org/10.1016/j.jecp.2022.105587

Qetrani, S., Ouailal, S. & Achtaich, N. (2021). Enhancing Students' Conceptual and Procedural Knowledge Using a New Teaching Approach of Linear Equations Based on the Equivalence Concept. *EURASIA Journal of Mathematics*, Science and Technology Education, *17*(7), 1-17, em1978. https://doi.org/10.29333/ejmste/10938

Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology, 99*(3), 561–574.

Rittle-Johnson, B., Schneider, M. & Star, J. (2015). Not a One-Way Street: Bidirectional Relations Between Procedural and Conceptual Knowledge of Mathematics. *Educational Psychology Review, 27*(4), Springer Science+Business Media New York. DOI 10.1007/s10648-015-9302-x

Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge in mathematics. In R. Cohen Kadosh & A. Dowker (Eds.), Oxford handbook of numerical cognition (pp. 1102-1118). Oxford, UK: Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199642342.013.014

Rittle-Johnson, B., Siegler, R. & Alibali, M. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346-362.

Schneider, M., & Stern, E. (2010). *The Developmental Relation Between Conceptual and Procedural Knowledge: A Multi Method Approach. Developmental Psychology, 46*(1), 178-192.

Star, J. (2005). Reconceptualizing Procedural Knowledge. *Journal for Research in Mathematics Education, 36*(5), 404-411.

Zulnaidi, H., & Zakaria, E. (2010). The effecte of information mapping stratigy on mathematics conceptual knowledge of junior high school students. *US-China Education Review, 7*(1), 26-31.

Zuya, H. (2017). Prospective Teachers' Conceptual and Procedural Knowledge in Mathematics: *The Case of Algebra. American Journal of Educational Research, 5*(3), 310-315. DOI:10.12691/EDUCATION-5-3-12

نموذج تدريسي مقترح لتحقيق التوازن في تقديم المعرفة المفاهيمية والإجرائية عند تدريس الرياضيات

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الملخص

الأهداف: تسعى هذه الدراسة إلى تقديم نموذج تدريسي مقترح لتحقيق التوازن بين المعرفة المفاهيمية والإجرائية. **المنهج:** اعتمد الباحثان على المنهج الوصفي التحليلي من خلال الاطلاع على العديد من الأدبيات والدراسات ذات العلاقة بالمعرفة المفاهيمية والإجرائية عند تدريس الرياضيات والتوازن بينهما، وتحليلها واستقراء نتائجها، كما تم استخدام المنهج الوصفي المسحي. **النتائج:** أظهرت نتائج الدراسة أن معلمي الرياضيات لا يهتمون بتحقيق التوازن بين المعرفة المفاهيمية والإجرائية، وغالباً ما يركزون على المعرفة الإجرائية عند تدريسهم للرياضيات. **الخلاصة:** قدمت الدراسة نموذجاً تدريسياً إجرائياً يساعد معلمي الرياضيات على تحقيق التوازن بين المعرفة المفاهيمية والإجرائية يستند إلى بعض نظريات التعلم والأسس العلمية تكون من ثلاث مراحل تدريسية متتابعة بشكل خوارزمي تتضمن تحقيق التوازن بين المعرفة المفاهيمية والإجرائية في كل مرحلة.

الكلمات المفتاحية: نموذج مقترح، المعرفة المفاهيمية، المعرفة الإجرائية، التوازن، تدريس الرياضيات.

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