A MODEL FOR DETERMINING OPTIMAL PRODUCT LINE COMBINATIONS: AN EMPIRICAL STUDY FOR KUWAITI INSURANCE MARKET

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Introduction

In this paper certain concepts will be discussed that are important for determining an optimal product line combination for a multiple line property-liability, or a sub-products for a life insurance company. In particular expected value-variance approach and its explination will be discussed in brief. It is the purpose of this paper to develop a business portfolio selection model consistent with insurance companies decision-making processes and the institutional constraints which they face in portfolio selection. The suggested model will be formulated in terms of a linear programming approach and related techniques and this will be shown to be equivalent to the E, σ^2 approach in certain respects. However, this paper relies extensively on theory and methodolgy developed in the field of finance

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for portfolio selection under conditions of uncertainty. Therefore, we shall try to apply the concepts of mean-variance efficient diversification to the selection of an optimal product mix for a multiple line insurance company or a sub-products for a life insurance company.

The use of mean-variance criterion in business portfolio selection is not intended to imply that such an approach actually is pursued by insurance companies. In other words, this is not intended to imply that insurance company managers consciously attempt to allocate premium volume among product lines according to the E, σ^2 criterion. Instead, the discussion is intended to support the hypothesis that such an approach is consistent with observed insurance company behaviors and hence can be used to describe and predict such behaviors.

1. DECISION-MAKING CRITERIA

A certain amount of premium is to be allocated among N product lines / sub-products of insurance protection. Let u_i refer to the underwriting profit for the i^{th} product line/sub-product and w_i refer to the proportion of the premium to be allocated for the i^{th} product line/sub-product. Clearly, $w_i \geq o$ and

$\sum_{i=1}^{N} w_i = 1$. Let W denote the vector of w_i which is called a business portfolio. The main object of the portfolio problem is to assess the performance of a given portfolio and to choose one portfolio from a given set of portfolios which is optimal. In order to assess the performance of given portfolio, one has to lay down the criteria of what constitute a good portfolio. We assume the vector of underwriting profit margins $u = (u_1, u_2, u_3, \ldots, u_N)$ is a random vector with the mean vector $U = (U_1, U_2, \ldots, U_N)$ and dispersion matrix $\Omega_{NxN} = \sigma_{ij}$, i.e. expected value of $u_i = Eu_i = U_i$, variance of $u_i = \sigma_{ii} = v_i$, covariance between u_i and $u_j = \sigma_{ij}$, where $i \neq j$ and $i,j = 1,2,3, \ldots, N$. This is quite reasonable to assume as the underwriting profit u_i on the i^{th} product line/sub-product is uncertain and thus can be assumed normally distributed random variables.

Let $W = (w_1, w_2, w_3, \ldots, w_N)$ be a business portfolio. The underwriting profit from W is defined as $W'u = \sum_{i=1}^{N} w_i u_i$ *. However, let W be a business fortfolio. The expected underwriting profit from W is defined by the formula:

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* The transpose of a vector b is denoted by b'.

$W'U = \sum_{i=1}^{N} w_i U_i$ and is denoted by Eu(W). And, the risk of W is defined by formula:

$W' \Omega W = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \sigma_{ij} w_j$ and is denoted by $\sigma^2(W)$.

In determining the optimal product line combinations an assessment of the performance of a given combination W is carried out based on the values of Eu(W) and $\sigma^2(W)$. The implications of the definition of Eu(W) are intuitively obvious and the more the value of Eu(W) the more desirable W is. The interpretation of $\sigma^2(W)$ is not immediately obvious. The variance of the underwriting profit from W, i.e. the variance of $W'u = \sum_{i=1}^{N} W_i u_i$, is equal to $\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} = \sigma^2(W)$. The smaller the value of the variance, the more the value of $W'u$ is concentrated around its expected value $W'U$ = Eu(W). Consequently, the smaller the variance, the more sure we are of getting Eu(W) when we employ W. In other words, the smaller the $\sigma^2(W)$ the more desirable W is.

Let W be a business portfolio. W is said to be the best if for every other business protfolio, say, B, one has: $Eu(W)>Eu(B)$ and $\sigma^2(W)<\sigma^2(B)$. If a best portfolio exists the portfolio selection problem

becomes simple. Unfortunately, except in trivial circumstances, best portfolios do not exist. This is mainly due to the fact that it is almost impossible to minimize an objective function and maximize another objective function simultaneously. Therefore, some compromise between both criteria is needed to choose portfolios which are optimal.

2. MEAN-VARIANCE APPROACH

This study demonstrates the linear programming portfolio selection model of combining product mix for an insurance company so as to minimize portfolio risk for a given required underwriting profit for the portfolio. The actual number of product lines/sub-products is not a revelant consideration in measuring portfolio diversification. If the correlation among product lines/sub-products is perfect and positive then naive diversification cannot reduce portfolio risk. Only in situations where the correlation is negative can naive diversification reduce portfolio risk.

The concept of portfolio diversification as it is presented in protfolio theory by Markowitz extends beyond naive diversification. He introduced efficient portfolios. Let W be a business portfolio. It is said to be efficient if :ι $Eu(W) \geq Eu(B)$ and $\sigma^2(W) \leq \sigma^2(B)$,for a business portfolio B.

Markowitz develops what is known as the critical line method which gives all efficient portfolios. He assumed that the variance-covariance matrix is non-singular in his method (Markowitz 1952).

In the next section we shall itroduce a model for determining optimal product mix for a multiple line-insurance company. However, although Markowitz approach was discussed in detail, we add here some more analysis. In the non-singular case a brief description of the mean-variance approach is given below.

Let $\lambda \geq o$ be a real number. Let ϕ_λ be the function defined on the set of all business protfolios defined by,

$$\phi_\lambda(W) = \lambda\, Eu(W) - \frac{1}{2}\, \sigma^2(W)$$

Suppose $\hat{W}$ is a business portfolio satisfying $\phi_\lambda(\hat{W})$ = max. $\phi_\lambda(W)$. Then $\hat{W}$ is efficient* and any efficient portfolio can be obtained in this way for some λ with the exception of the portfolio obtained by maximizing $W' U$ over all business portfolios

In particular, the business portfolio which maximizes $\phi_o(W)$ or, equivalently, the one which minimizes $\sigma^2(W)$ is called Minimum Variance Portfolio and is denoted by MVP. **MVP is given by (press 1972), and it is explained unmerically by (Francis 1976).**

3. A LP MODEL FOR DETERMINING OPTIMAL PRODUCT MIX

We already know that in order to find efficient portfolios one can maximize:

$$\phi_\lambda W = \lambda \sum_{i=1}^{N} w_i U_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

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* One can prove this proposition as follows: Let $B \epsilon IP$ be a portfolio such that, $\hat{W}' U \leqslant B' U$ and $\hat{W}' \Omega \hat{W} \geqslant B' \Omega B$. We have to show that $\hat{W}' U = B' U$ and $\hat{W}' \Omega \hat{W} = B' \Omega B$. Suppose $\hat{W}' U < B' U$. Then $\lambda \hat{W}' U < \lambda B' U$ since $\lambda > o$. Further, $-\frac{1}{2} \hat{W}' \Omega \hat{W} \leqslant -\frac{1}{2} B' \Omega B$. $\phi_\lambda \hat{W} < \phi_\lambda B \leqslant \phi_\lambda \hat{W}$, a contradiction. Suppose, $\hat{W}' U = B' U$ but $\hat{W}' \Omega \hat{W} > B' \Omega B$. This again gives a contradiction.

This problem falls into the realm of quadratic programming. In this section, we propose a method of solving this quadratic programming problem by linear programming techniques.

Consider the function Ψ defined on IP by

$$\Psi_{W\in IP}(W) = W' \Omega W$$

it is a continuous function on IP. Let V_o = Min.$_{W\in IP}$ $\Psi(W)$ and V_1 = max.$_{W\in IP}$ $\Psi(W)$. It is obvious that $0<V_o<V_1<\infty$. Also, $V_o = \sigma^2(MVP)$. Since IP is a compact connected subset of the n-dimentional space R^n, every value in the interval (V_o, V_1) is attained by some portfolio $W\in IP$.

Let $\hat{W}\in IP$ be such that $\hat{W}' U = \max_{W\in IP} W' U$ = max.$_{W\in IP}$ Eu(W), we call this portfolio a maximum underwriting profit portfolio (Max. UP). However, one can easily find (max. UP).

One can propose that, in case $U_k = \max_{1\leq i\leq N} U_i$ the business portfolio $\hat{W}' = (o,o,\cdot\cdot,1,\cdot\cdot,o,o)$, where unity in the k^{th} position, is a maximum underwriting profit portfolio and its risk is

σ_{kk}^{2} *.

There are instances of systems where none of the maximum underwriting profit portfolios which are pure is efficient. We say that a portfolio $(w_1, w_2, w_3, \ldots\ldots, w_N)$ is pure if $w_i = 1$ for some i. In other words, a pure portfolio is one where the entire amount of premium is allocated to one business line (mainly with higher profit). The following is an example: Consider a system of N product lines/sub-products with $U' = (g,g,g,g,\ldots\ldots,g)$ and $\Omega = I_N$, the identity matrix with ones in the diagonal and zeros in the non-diagonal positions, where $g>o$ is constant. Any portfolio of the form $(o,o,o,\ldots\ldots,1,\ldots,o,o,o)$ is maximum underwriting profit portfolio with expected profit g and risk 1. But the portfolio $(\frac{1}{N}, \frac{1}{N}, \frac{1}{N}, \ldots\ldots\ldots\ldots, \frac{1}{N})$ has expected underwriting profit g and risk $\frac{1}{N}$ and consequently is better than any of the pure portfolios.

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* This proposition can be proved as follows: Let $Y' = (Y_1, Y_2, Y_3, \ldots\ldots\ldots, Y_N)$ be any other business portfolio. Then $Eu(Y) = Y'U =$

$$\sum_{i=1}^{N} Y_i U_i \leq \sum_{i=1}^{N} Y_i \left(\max_{1\leq j\leq N} U_j\right) = \sum_{i=1}^{N} Y_i U_k$$

$$= U_k \left(\sum_{i=1}^{N} Y_i\right) = U_k = Eu(\hat{W}).$$

Therefore, we propose that there exists at least one portfolio with maximum underwriting profit which is efficient*.

The objective of obtaining an optimal product line combination of risk no greater than a prescribed number and based on mean-variance efficient diversification can be specified in matrix notation as the following mathematical programming problem:

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* One can prove this proposition as follows: Let E be the collection of all portfolios with maximum underwriting profit. This set is non-empty as we can always find a pure portfolio with maximum underwriting profit. We claim that E is a closed subset of R^N. Let W^k, $k \geq 1$ be a sequence in E converging to $\hat{W}$. We have to show that $\hat{W} \in E$.

Let $W^{(k)} = (w_1^k, w_2^k, w_3^k, \ldots\ldots, w_N^k)$. Eu $w^{(k)} =$

$\sum_{i=1}^{N} w_i^k U_i = g$, constant for every $k \geq 1$. But

$\sum_{i=1}^{N} w_i^{(k)} U_i$, $k \geq 1$ converge to $\sum_{i=1}^{N} \hat{w}_i U_i = g$.

Hence, $\hat{W} \in E$. Since E is a closed subset of the compact set IP, E is also compact. Consequently, minimum $W' \Omega W$ exists and is attained by a vector $Y \in E$. Thus Y has maximum expected underwriting profit and minimum risk among such vectors and hence is efficient.

Let : $V_o = \text{Min.}_{W \epsilon IP} \acute{W} \Omega W$

$V_1 = \text{Max.}_{W \epsilon IP} \acute{W} \Omega W$

$\hat{V}$ = real number lying between V_o and V_1

$\hat{W}$ = solution to the business portfolio problem

The Model:

Maximize : $U_i w_i$

Subject to : $\Omega w_i \leq \hat{V} e_N$

$A w_i \geq B$

$w_i \geq o$

$\sum_{i=1}^{N} w_i = 1$

where,

W = the vector of proportions allocated to each product line

U = the vector of expected underwriting profit margins for each product line

Ω = the covariance matrix of underwriting profit margins among product lines.

$\hat{V}$ = the acceptable level of risk within the insurance portfolio.

A = a matrix of constraint coefficients.

B = a vector of constraint constants indicating the upper and lower limits applicable to each constraint, levels of proportions of total premiums that can be allocated to the product lines.

Then $\hat{W}$ is an efficient portfolio of risk $\leq \hat{V}$.*.

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* The proof is carried out in the following steps:

(1) We first claim that a feasible solution to the above linear programming problem exists. For, let $Y \varepsilon IP$ be such that $Y'\Omega Y = \min._{W \varepsilon IP} W'\Omega W = V_o$. By proposition $\sigma_i^2(Y)$ = risk of the i^{th} product when Y is used $= \sum_{j=1}^{N} \sigma_{ij} Y_j$

$= V_o \leq \hat{V}$. Consequently, Y satisfies all the constraints imposed in the above contribution. Hence, Y is a feasible solution to this problem.

(2) We next claim that $\sigma^2 \hat{W} \leq \hat{V}$. For $\sigma^2 \hat{W} = \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{w}_i \hat{w}_j \sigma_{ij} = \sum_{i=1}^{N} (\sum_{j=1}^{N} \sigma_{ij} \hat{w}_j) \hat{w}_i \leq$

$\sum_{i=1}^{N} \hat{V} \hat{w}_i = \hat{V} \sum_{i=1}^{N} \hat{w}_i = \hat{V} \times 1 = \hat{V}$.

(3) Finally, we show that $\hat{W}$ is efficient. Let $S = \sigma^2 \hat{W} = \hat{W}' \Omega \hat{W} \leq \hat{V}$. Let D be the collection of all portfolios of risk S. It is clear that among these portfolios $\hat{W}$ has maximum expected underwriting profit. Hence, $\hat{W}$ is efficient.

4. AN EMPIRICAL STUDY IN MATHEMATICAL PROGRAMMING OF BUSINESS PORTFOLIO SELECTION

Having developed a linear programming portfolio selection model in the field of business, we need to test it using some real data. Our investigation was designed to measure the actual portfolio with a view to developing a base for assessing future efficient portfolios. The purpose here is not to test the business decisions of the past only, but to put forward a new procedure that could be followed in the future.

(i) Formulating an empirical test:

Since an investigation into the experience of an actual case study could yield useful information, real data have been collected in order to test the LP busines portfolio selection model. For every product line included in the investigation a fifteen years record has been collected for analysis. Our portfolio selection based on past performances alone, assumes that average underwriting profits of the past are agood estimate of the "Likely" profit margins in the future, and variability in underwriting profits in the past is a good measure of the uncertainty of profit margins in the future. However, although the computer has been used to calculate expected underwriting profits, variances and covariances, a lot of hand work has been done in order to furnish the original data for analysis.

(ii) Description and development of input data for the case study:

We shall assume there is a LP portfolio selection model which provides the insurer with the highest expected underwriting profit subject to a certain risk level. Although the assumptions underlying the model may not always be entirely realistic, it provides great computational advantages in that it eliminates the need for using the non-linear equations in formulating a portfolio selection model and consequently using linear programming instead of quadratic programming.

To test the model, data were collected for the entire Kuwaiti insurance market (eighteen insurance companies, for a period of fifteen years commencing in 1970). For each company, each year, and each of seven lines of insurance, data for a number of accounting and underwriting variables were collected.*

Roughly speaking, the underwriting profit each year for a product line was defined to be the premiums earned minus (loss incurred plus other underwriting expenses incurred). Table (1) shows the average underwriting profit margins for all seven product lines in the Kuwaiti insurance companies . The underwriting profit of each line was assumed to be governed by one probability distribution.

The annual underwriting profit for each line in the last fifteen years is then used to calculate the arithmetical mean.

The average is approximate, of course, since the data may have been affected by the lack of more original historical data. However, the results are presented in the following table :

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*The following items were obtained: earned premiums, loss incurred, other underwriting expenses, incurred commissions, expenses. However, the entire Kuwaiti insurance market is defined as all property-liability business presented by companies included in the 1984 Kuwaiti annual report (Five domestic, six arab and seven foreign companies).

Table (1)

The Expected Average of Underwriting Profit/Loss Margins of Product Lines in Kuwaiti insurance companies 1970-1984

No.	Line of Business/Code Name	Expected Average of Underwriting P/L* %
1	Fire Insurance / FI	(2.6)**
2	Marine & Aviation / MA	31.3
3	Automobile Bodily injury and Property damage Liability / ABPL	(14.2)
4	Automobile Property / AP	13.9
5	Worker's Compensation / WC	22.5
6	Engneering Insurance / EI	131.7
7	Miscellaneous Bodily Injury / MBI	60.6

* P/L means underwriting profit-loss margins
** Number between parantheses correspond to loss margins.

Because business portfolio selection should consider underwriting profit/loss margins and the effect of net premiums of product lines on net profits and losses, some more calculations were performed so that the estimated parameters (profits/losses and covariance matrices) would be more effective. The effect of net premiums may be considered an objective as well as underwriting profit/loss margins because the investment of net premiums is almost used to improve the net result of several product lines. In addition, ignoring the effect of earned premiums may distort the business policy of companies which have allocated a high proportion of their premiums to third party insurance, for example, they will feature as having been less profitable in any comparison with those that have concentrated on engneering insurance. The weighted average of underwriting profit/loss margin that represents the cornerstone of business portfolio selection may be defined ; underwriting profit-loss multiplied by net earned premium of each product line/total net earned premiums of business lines.*

Hence, the objective function is expressed as maximize, U, where U stands for total weighted average of business profit/loss margins from both the under writing profit/loss and the effect of earned premium on these margins. Table (2) indicates the weighted average of expected underwriting profit/loss margins of product lines.

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* For example, the weighted average of underwriting profit/loss margins for fire insurance product lines is;

$$= \text{P/L (fire)} \times \frac{\text{NEP (fire)}}{\text{NEP (total business)}}$$

Table (2)

The Weighted Average of Expected Underwriting Profit/Loss Margines of Product Lines in Kuwaiti insurance companies 1970-1984

No.	Line of Business / Code Name	Weighted Average of Expected underwriting P/L %
1	Fire Insurance / FI	.04
2	Marine & Aviation / MA	3.88
3	Automobile bodily injury and Property damage liability / ABPL.	(2.90)
4	Automobile Property / AP	8.7
5	Worker's Compensation / WC	2.8
6	Engneering Insurance / EI	.44
7	Miscellaneous Bodily Injury/ MBI	1.9

According to the model and the real data presented in table (1) and table (2), the underwriting manager's objective is to allocate total premiums among the business lines so as to maximize the weighted average of expected underwriting profit margin or minimize the weighted average of expected underwriting loss margin of product lines subject to (i) an acceptable level of risk and (ii) a set of linear constraints which restrict the proportions of total premiums which can be committed to any particular business line. However, the allocation pertains to the portion of total premium volume that is allocated to the various business lines regardless of the actual amount of total premiums.

More specifically, the figures published in the above tables and the tables in Appendix A and Appendix B were used in determining efficient business portfolios using the data for the period 1970/1984 inclusive.

5. RESULTS AND ANALYSIS

The business portfolio selection model is solved twice for the type of business included in the study. In the first solution the allocation is unconstrained. In other words, neither upper nor lower bounds are placed on the allocation of premiums to any product line. In the second solution the allocation is constrained. For each company the constraints on some business lines are the company's historical upper or lower business line allocations. The procedure used in this study is to assume that each company is inforced by law to underwrite some particular business such as automibile liabilities

In addition insurance companies in real life do not ignore complementarily among product lines. For example, the two automobile covarages ABPL and AP have some relation to one another in the amount of premiums that can be allocated to either line of business. However, no company has complete discretion in its premium allocation but every company is able to obtain allocation limits in the future that were attained in the past.

In the light of that our purpose is not to compute optimal business portfolios which conform exactly to reality, the optimization results for the best combination of business lines are presented in table (3). For both constrained and unconstrained optimization this table presents expected weighted average of underwriting profit margins and risk levels. Also, the results in table (3) emphasize the impact of constraints upon obtainable expected underwriting profit margins and risk.

Table (3)

Optimal Business Line Allocations

for An Insurance Company

Business Line	Unconstrained Combination			Constrained Combination	
	1	2	3	4	5
Fire/FI				.04	
Marine/MA					
Atom. Liability/MA	.2	.05		.32	.12
Atom. Property/AP	.8	.95	1	.64	.87
Wor. Compensation/WC					
Engn. Ins./EI					.01
Mise.Bodily/ Inj./MBI					
Risk ≤	.2	.3	.5	.1	.25
Max. Underwriting profit	.064	.081	.087	.046	.072
Constraints on Product lines				$W_1 \geq .04, W_3 \geq .25$ $W_5 \leq .20$	$W_1+W_2 \leq .12$ $W_3+W_4 \geq .60$ $W_6 \geq .01$

Also, table (3) shows that Automobile liability product line, which yields a negative profit margins, is included in unconstrained portfolios in particular when risk levels go down. This inclusion indicates that it is not desired and is included only because its contribution to the variability of portfolio underwriting profit margins is relatively small.

The results presented in tables (3) and (4) show the various possible efficient portfolios available to the business manager. Such a manager can easily decide which efficient portfolio will be the one which coincides with his attitude towards risk. If he is aggressive he would choose portfolio No.3 (risk level = .336 and underwriting profit = .087), however, a conservative, manager can select portfolio No. 4 (risk level = .068 and underwriting profit = .046). In addition, there are some other efficient portfolios available to the business manager. However, as he has some knowledge of his risk-underwriting profit indifference function, the optimum combination of business lines can easily be selected.

Table (4)

Optimal Expected Underwriting
Profit Margins and Risk for
Five Portfolios Based on Historical Data

Portfolio	Minimum Risk	Maximum Underwriting Profit
1	.132	.064
2	.270	.081
3	.336	.087
4	.068	.046
5	.199	.072

The unconstrained minimum risk optimization results can be useful to a company by indicating which of its product lines (those absent from the portfolio) might be candidates for reinsurance. The maximum profit margin results in the unconstrained case are not useful. These results imply that companies should become specialty companies, generally emphasizing one line of insurance, Atomobile Property. However, the overall expected underwriting profit margin can be decreased along the efficient frontier with a slight decrease in the insurer's risk.

6. SUMMARY AND CONCLUSIONS

The primary purpose of this paper was to present a plausible model for determining the optimal product line combination of a multiple line insurance company. The model assumes that the objective function is to maximize the expected weighted average of underwriting profit margin subject to some permissible level of risk. However, the former portfolio selection models include risk as a measured variable in a manner which was represented as quadratic linear programming function. In this study the risk variable has been formulated as a linear constraint on a miximization of underwriting profit criterion. The suggested model provides an efficient approach to the selection of optimal business portfolios which can be followed mechanically without resort to professional management.

Two sets of optimal product line combinations were computed. In the constrained and unconstrained solutions the computations were based upon the observed expected value of underwriting profit margins and covariances of profit margins for each line of business. In the constrained portfolios the constraints were based upon past premiums allocations for each line of business. Also, portfolio managers should prescribe risk levels for their business portfolios.

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APPENDIX A

OVERALL UNDERWRITING RESULTS
RETAINED BUSINESS IN KUWAIT
GENERAL INSURANCE MARKET
(DOMISTIC, ARAB, AND FOREIGN COMPANIES)

<u>APPENDIX A</u>

Table (A-1): Weighted Average of Exptected Profit or Loss (Fire)

OOO' D.K.

Premiums & Ratios	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
Gross Earned Premiums / GEP	118	136	176	180	215	265	285	265	385	432	655	623	718	806	902
Incurred Loss / IL	50	80	59	96	192	228	140	209	242	730	315	837	1426	460	654
Underwriting Expenses Incurred / UEI	19	22	28	26	36	48	51	77	81	92	143	140	154	177	231
Net Earned Premiums / NEP	99	114	148	144	179	217	234	288	304	340	423	482	564	629	671
Net Underwriting Profit or Loss/NUP (L)	49	34	89	48	(13)	(11)	94	79	62	(390)	108	(355)	(862)	169	17
Raito of NUP (L) to NEP %	49	29	60	33	(7)	(5)	40	27	21	(115)	25	(73)	(153)	27	3
Weight Ratio of NUP (L) to NEP%	1.7	1	2.3	1	(.2)	(.2)	1	.6	.5	(2.6)	.6	(1.9)	(4.2)	.9	.1
Weighted Average Ratio of NUP (L) to NEP %	.04														

APPENDIX A

Table (A-2): Weighted Average of Exptected
Profit or Loss (Marine)

000' D.K.

Premiums & Ratios	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
Gross Earned Premiums / GEP	742	776	893	1190	1758	2114	2626	3066	2455	2255	2729	2434	2315	2269	2174
Incurred Loss / IL	462	309	406	849	1437	1543	2115	3448	1423	562	1034	1049	730	529	533
Underwriting Expenses Incurred / UEI	117	145	196	265	401	482	578	631	458	413	513	436	467	450	476
Net Earned Premiums / NEP	625	631	697	925	1357	1632	2049	2436	1997	1842	2216	1998	1848	1819	1698
Net Underwriting Profit or Loss/NUP (L)	163	322	291	76	(80)	89	(67	(1013)	574	1280	1182	949	1117	1290	1165
Raito of NUP (L) to NEP %	26	51	42	8	(6)	6	(3	(42	29	70	53	48	60	71	54
Weight Ratio of NUP (L) to NEP%	5.8	10.3	7.7	1.6	(1.5)	1.4	(.6	(8.2	4.1	8.6	7	5.3	5.5	6.7	4.5
Weighted Average Ratio of NUP (L) to NEP %	3.88														

APPENDIX A

Table (A-3): Weighted Average of Exptected Profit or Loss (Automobile Liability)

000' D.K.

Premiums & Ratios	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
Gross Earned Premiums / GEP	1000	1032	1275	1563	1628	1645	1970	2584	3011	3243	3738	4276	5807	6679	6778
Incurred Loss / IL	600	700	694	912	1212	1117	1343	2307	3408	4472	4045	5264	6581	8349	7832
Underwriting Expenses Incurred / UEI	247	235	333	405	430	392	430	546	652	699	707	863	1346	2105	1560
Net Earned Premiums / NEP	753	779	942	1158	1198	1253	1540	2038	2359	2544	3031	3413	4461	4574	5218
Net Underwriting Profit or Loss/NUP (L)	153	79	248	246	(14)	136	197	(269	(1049)	(1925)	(1014)	(1851)	(2120)	3775	2614
Raito of NUP (L) to NEP %	20	10	26	21	(1)	11	13	(13)	(44)	(76)	(33)	(54)	(48)	(83)	(50)
Weight Ratio of NUP (L) to NEP%	5.2	2.5	6.5	5.2	(.04)	2	2.1	(2.1)	(7.4)	(13)	(6)	(10)	(10.5)	(19.8)	(12.9)
Weighted Average Ratio of NUP (L) to NEP %	(2.9)														

APPENDIX A

Table (A-4): Weighted Average of Exptected
Profit or Loss (Automobile Property)

000' D.K.

Premiums & Ratios	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
Gross Earned Premiums / GEP	1203	1579	2153	2654	2996	4105	5806	7434	8541	9286	9605	10819	1261[illegible]	1195[illegible]	10014
Incurred Loss / IL	1204	1207	1765	1806	2105	2885	4229	5049	5547	6191	5444	5567	610[illegible]	525[illegible]	4016
Underwriting Expenses Incurred / UEI	298	386	562	688	792	977	1268	1570	1851	2002	1816	218[illegible]	292[illegible]	3767	2304
Net Earned Premiums / NEP	905	1193	1591	1966	2204	3128	4538	5864	6690	7284	7789	863[illegible]	969[illegible]	818[illegible]	7710
Net Underwriting Profit or Loss/NUP (L)	(229)	(14)	(174)	160	99	243	309	815	1143	1093	2345	3069	358[illegible]	292[illegible]	3694
Raito of NUP (L) to NEP %	(33)	(1)	(11)	8	5	8	7	14	17	15	30	36	37	3[illegible]	37
Weight Ratio of NUP (L) to NEP%	(10.6)	(.4)	(4.6)	3.4	2	3.5	3.3	8.4	8.1	7.8	14	17	17.[illegible]	15.[illegible]	14.1
Weighted Average Ratio of NUP (L) to NEP %	8.7														

APPENDIX A

Table (A-5): Weighted Average of Exptected Profit or Loss (Worker's Compensation)

000' D.K.

Premiums & Ratios	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
Gross Earned Premiums / GEP	352	354	407	455	662	903	1423	2163	2984	3183	3324	3814	4072	4674	5224
Incurred Loss / IL	251	241	203	218	370	446	784	1089	1712	1850	1927	2066	2872	3038	3353
Underwriting Expenses Incurred / UEI	87	87	106	118	175	215	311	457	647	686	629	769	944	1473	1202
Net Earned Premiums / NEP	265	267	301	337	487	688	1112	1516	2337	2497	2695	3045	3128	3201	4022
Net Underwriting Profit or Loss/NUP (L)	14	26	98	119	117	242	328	427	625	647	768	979	256	163	669
Raito of NUP (L) to NEP %	5	10	33	35	24	35	29	28	27	26	28	32	8	5	13
Weight Ratio of NUP (L) to NEP%	.5	.9	2.6	2.5	2.1	3.4	3.3	3.9	4.5	4.3	4.5	5.3	1.2	.8	2.6
Weighted Average Ratio of NUP (L) to NEP %	2.8														

APPENDIX A

Table (A-6): Weighted Average of Exptected
Profit or Loss (Engnerring Insurance)

000' D.K.

Premiums & Ratios	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
Gross Earned Premiums / GEP	34	11	14	15	23	28	71	129	122	84	65	74	85	98	149
Incurred Loss / IL	10	39	66	(34)	29	8	12	29	47	54	37	52	14	41	62
Underwriting Expenses Incurred / UEI	8	3	4	4	6	7	16	27	26	18	12	15	20	31	34
Net Earned Premiums / NEP	26	8	10	11	17	21	55	102	96	66	53	59	65	67	115
Net Underwriting Profit or Loss/NUP (L)	16	(31)	(56)	45	(12)	13	43	73	49	12	16	7	51	26	53
Raito of NUP (L) to NEP %	62	(388)	(560)	409	70	62	78	72	51	18	30	12	79	39	46
Weight Ratio of NUP (L) to NEP%	.6	(1)	(1.5)	1	(.2)	.2	.3	.6	.3	.1	.1	.04	..3	.14	.26
Weighted Average Ratio of NUP (L) to NEP %	.44														

APPENDIX A

Table (A-7): Weighted Average of Exptected Profit or Loss (Miscellaneous Podily Injury)

000' D.K.

Premiums & Ratios	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
Gross Earned Premiums / GEP	202	188	132	144	147	188	307	377	395	451	631	772	786	1067	1078
Incurred Loss / IL	20	20	29	66	42	72	51	98	79	179	210	233	424	384	390
Underwriting Expenses Incurred / UEI	50	46	34	37	39	45	67	80	86	97	119	156	203	336	248
Net Earned Premiums / NEP	152	142	98	107	108	143	240	297	309	354	512	616	583	731	830
Net Underwriting Profit or Loss/NUP (L)	132	122	69	41	66	71	189	199	230	175	302	383	159	347	440
Raito of NUP (L) to NEP %	87	86	70	38	61	50	79	67	74	49	59	62	27	47	53
Weight Ratio of NUP (L) to NEP%	4.7	3.9	1.8	.9	1.2	1	1.9	1.6	1.6	1.2	1.8	2.1	.8	1.8	2.2
Weighted Average Ratio of NUP (L) to NEP %	1.9														

APPENDIX B

VARIANCE- CORARIANCE MATRIX
UNDERWRITING PROFIT/LOSS
GENERAL INSURANCE MARKET

APPENDIX B

Table (1): Variance-Corariance Matrix

Variables	F I	M A	ABPL	A P	W C	E I	MBI
FI	.029	- .008	.080	- .046	- .005	.004	.008
MA	- .008	.221	- .088	.060	- .016	.002	.018
ABPL	.080	- .088	.723	- .349	- .001	.022	.202
A P	- .046	.060	- .349	.336	.008	- .012	- .002
WC	- .005	- .016	- .001	.008	.203	- .002	- .007
EI	.004	.002	.022	- .012	- .002	.002	.001
MBI	.008	.018	.020	- .002	- .007	.001	.012