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A MATHEMATICAL PROGRAMMING MANAGEMENT MODEL FOR A STORAGE REPLENISHMENT OCEAN TRANSPORTATION PROBLEM

Key Words

Mixed-integer Programming; Ship scheduling; Transportation; Optimization; CPLEX.

Abstract

This paper presents a mathematical programming model for a scheduling transportation problem. We examine a specific demand scenario that faces many oil companies, based on which a company aims to satisfy a given client's demand requirement of crude oil at a minimum cost. The overall cost of transportation is comprised of daily operational expenses of vessels, cost of time and spot chartering, and penalties incurred on violating specified delivery time windows. Due to the high cost associated with time and spot chartering, it is lucrative to utilize self-owned vessels, to the extent possible, and resort to chartered vessels as needed. A solution of the developed model specifies dispatching and delivery days of shipments and determines the fleet mix of vessels, and hence the model can be utilized for scheduling, planning, managing, and negotiating purposes.

Introduction

Routing and scheduling of vessels is an elaborate and significant level of planning of fleet management in any maritime transportation system. It involves assigning shipments to vessels, selecting delivery dates, determining the required fleet size mix consisting of company-owned and chartered vessels of different types, checking whether it is lucrative to perform a spot charter for another operator, and so on. Because of the high cost associated with operating and chartering vessels, efficient routing and scheduling of vessels has

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the potential of enormous saving in the overall fleet cost. This is particularly true when realizing that a typical vessel in a merchant fleet usually costs millions of US dollars, and the daily operational cost of a vessel amounts to tens of thousands of US dollars (Ronen 1983,1993).

Problem Specifics

In the specific problem that is the focus of this paper, a product is to be transported from a single source to a single destination based on agreed upon contracts that specify delivery time intervals and related penalties. Shipments that are not delivered within the specified delivery time intervals receive penalties agreed upon between the transporting company and the client. The transporting company owns a number of vessels (company-owned), but they are usually not adequate to satisfy demand requirements of the client; hence the company resorts to time and spot chartered vessels as needed. Accordingly, the fleet of vessels is typically composed of company-owned and chartered vessels. There are different types of vessels; each is characterized by its capacity, size, speed, daily operational expenses, chartering expenses and so on, and thus two vessels of the same type are assumed to have similar characteristics. The client specifies the total demand of the product over some specified time horizon, which is in turn partitioned into a number of smaller quantities (subdemands), each of which is associated with a feasible delivery time interval. The subdemands are usually specified by the minimum and maximum allowable quantities of the product to be shipped to the client, which are determined based on the client's storage facilities and the anticipated consumption of the product. Therefore, the replenishment of the client's storage facilities is driven by the overall demand of the product and some stringent delivery time-windows associated with the subdemands. A shipload that is not delivered within any of the specified delivery time-intervals receives a penalty agreed upon by the client and the transporting company as will be discussed later in the paper. This particular demand structure is faced by many oil companies where crude oil needs to be transported from a source to a destination according to the above prescribed demand structure. A follow-up research examines another demand structure that is also faced by oil companies, namely, when the demand structure is specified based on the daily consumption rates at the destination and the penalties associated with certain levels of the destination's storage facility.

Related Literature

Water transportation is of special significance to many oil companies in the world, where vessels mainly move crude oil and its related products. For the United States, for example, which is the world's largest energy consumer, seaborne petroleum imports are an essential lifeline to the oil deposits of the Middle East, East Asia, and Latin America (Warf, 1989). Efficient vessel transportation systems could lead to significant savings due to the high cost associated with vessel scheduling. Vessel routing and scheduling problems have attracted many researchers in the past; however, scheduling problems related to other means of transportation such as vehicle, rail, and airline have attracted considerably more research effort (Ronen, 1983, 1993). Although a wide range of quantitative approaches has been employed to tackle vessel scheduling problems, many of these problems still pose challenges to researchers because of the complexity and high degree of uncertainty involved in vessel operation. This research is intended to highlight the importance of efficient scheduling of vessels in the overall fleet operation. In particular, the proposed modeling approach indicates that appropriate scheduling can significantly reduce the overall cost.

Dantzig and Fulkerson (1954) studied a tanker scheduling problem of a homogeneous fleet, i.e., carrying capabilities, speeds, and operating expenses were the same for all vessels. Loading and discharging dates were predetermined. The objective was to minimize the number of vessels to meet a specified schedule. One loading and one discharging port per vessel voyage were assumed. Briskin (1966) extended the problem of Dantzig and Fulkerson (1954) by allowing several discharging ports. He described a heuristic clustering procedure in which unloading ports were clustered together and used a transportation method to schedule the tankers. The author utilized dynamic programming to determine the schedule of each ship through its cluster of unloading ports.

Bellmore *et al.*, (1968) presented a modification of the Dantzig and Fulkerson (1954) problem, in which there was an insufficient number of tankers available to satisfy the demand requirements. Delivery "utilities" were presented to determine which deliveries to cancel (Miller, 1987). In another paper, Bellmore *et al.*, (1971) extended Dantzig and Fulkerson's (1954) problem by allowing various types of vessels and permitting partially loaded vessels. Delivery dates were specified within a certain time interval. The objective was to maximize the total utility of deliveries. The utility of a delivery reflected its desirability and cost. Negative utilities were associated with empty legs. The problem was modeled as a mixed integer linear program. The authors suggested employing decomposition with a branch and bound algorithm and network subproblems. Nevertheless, no applications or results were given.

Laderman *et al*., (1966) studied a ship routing problem that might be faced either by a tramp shipping company or by an industrial operator. Specified quantities of bulk commodities were to be shipped between specific pairs of ports. The fleet considered was composed of vessels of different characteristics. The objective was to minimize the number of ships needed to meet the shipping requirements. This problem was modeled as a linear program, which assigns vessels to trips, while rounding any resulting fractional number of trips. Rao and Zionts (1968) analyzed a similar problem. They considered whether the chartering option was needed. The objective was to minimize the chartering and operating expenses of vessels. The authors employed a column generation algorithm based on OUT-OF-KILTER sub-problems to reduce the size of the linear program.

Naslund (1970) examined the problem of delivering wood pulp from many loading ports to numerous discharging ports. This problem is faced by many companies in Northern Europe, where such companies need to determine vessel type, size, and speed. Companies must also efficiently determine suitable ports of call. The objective was to minimize the total transportation expenses from the port of origin to the final inland destination. The author found the problem to have certain characteristics similar to the warehouse location problem. Therefore, a warehouse location algorithm similar to the one proposed by Baumol and Wolf (1958) was employed to solve this problem. The algorithm provided a local minimum; however, it did not assure a global minimum.

Mathis (1972) investigated an oil supply system in which the decision variables were fleet size and mix, sizes of shore tanks, and routes of vessels. Transshipments were permitted at discharging ports. A number of assumptions were made to simplify the problem. Some of these assumptions were (a) only one type of crude oil was considered, (b) a tanker had to stay in its assigned route, (c) sizes of vessels were continuous, and (d) analytical approximations were used for the cost function. The objective was to optimize (minimize) the total cost of the system. The problem was modeled as a nonlinear integer program and the author exploited the problem's structure to solve it by a branch-and-bound procedure with generalized upper bounds. Flood (1954) attempted to determine optimum routes and schedules for the US military tanker fleet transporting bulk oil products worldwide. He assumed a given fleet size and minimized the overall distance traveled in ballast (empty) of the vessels by solving a transportation problem. McKay and Hartly (1974) considered the tanker scheduling problem of the US Defense Fuel Supply Center (DFSC) and the Military Sealift Command (MSC) in the worldwide distribution of bulk oil products. They permitted multiple loading ports in a voyage and attempted to minimize the operating expenses of the vessels and the cost of buying the products at the loading ports. The problem was modeled as a mixed-integer linear program and was solved as a linear program with a specialized rounding technique for the discrete variables.

Fisher and Rosenwein (1989) considered the efficient scheduling of a fleet of vessels involved in a pickup and delivery of bulk cargoes. In this model, a menu of candidate schedules for each ship was generated. Selecting an optimal schedule from this menu was modeled as a set packing problem and solved with a dual method. This model was programmed in Pascal on a VAX 8600 and was composed of three modules: (1) the schedule generator, (2) the set packing algorithm, and (3) a color-graphic interface that facilitates display and interactive modification of different schedules. The model was tested based on real-world data obtained from the Military Sealift Command of the US Navy. The resulting solutions indicated a potential for saving up to about $30 million per year over the manual system that was used.

Baker (1981) presented an interactive ship scheduling model for a situation which resembles the vehicle scheduling problem. The model provided schedules for distribution of oil products from a single refinery to several delivery points. A linear programming model and a network model were used to determine voyage selection and quantities of products shipped, respectively.

Now, we present research that is particularly related the problem of the present paper. For further details on vessel routing and scheduling problems and models, the reader may refer to Ronen (1983, 1993).

A vessel scheduling problem concerning the transportation of crude oil from the Middle East to Europe and North America was considered by Brown *et al.*, (1983), who formulated an integer-programming model and utilized column generation techniques to solve it. The demand structure was specified by a sequence of cargos that needed to be delivered during the planning horizon. The approach adopted by Brown, *et al.* (1983) is to generate a partial set of complete feasible schedules (in a column generation fashion), along with the generated schedules' costs, and then develop an elastic set partitioning programming model. The model developed in this paper differs from that model in that it incorporates different vessel types and combines the process of constructing and selecting feasible schedules within the model itself. Ronen (2002) proposed a mixed-integer programming model and a cost-based heuristic procedure for a vessel transportation problem. The approach adopted by Ronen (2002) divides the solution of the problem into two stages. The first stage determines the slate of shipments to be made, and the second stage generates vessel schedules. In contrast, the model developed in this paper determines the fleet size mix as well as the detailed vessel schedules. Hassan and Al-Yakoob (1999) considered a similar scheduling transportation problem, but it was with a much relaxed demand structure, whereby only an overall demand requirement was specified without any restrictions on the delivery time intervals.

Modeling Preliminaries

In this section, we present notation, assumptions, and a penalty function that will be used in the next section to formulate a mixed-integer program for the Storage Replenishment Ocean Transportation Problem prescribed in the previous section.

Notation and Assumptions

Consider the following list of notations.

- $h = 1, ..., L$:Indices for the days of the time horizon, where L denotes the duration of the time horizon.
- $H = \{1, ..., L\}$:The set of days of the time horizon.
- T : The total number of vessel types.
- $t = 1, ..., T$:Indices for vessel types.
- Ω_t :Capacity of a vessel of type t.
- $T_{1,t}$:Time required for a vessel of type t to travel from the source to the destination, which includes the time required to load the vessel in the source plus the time the vessel takes to reach the destination.
- $T_{2,t}$:Time required to unload a vessel of type t at the destination's facility plus the time this vessel takes to travel back to the source.
- $T_t = T_{1,t} + T_{2,t} = 2T_{2,t} = 2T_{1,t}$. (i.e., we assume that $T_{1.t} = T_{2,t}$).
- O_t :Number of company-owned vessels of type t.
- $O = \sum_{t=1}^{T} O_t$.
- CH_t: Number of available vessels of type t that can be possibly chartered.
- $CH = \sum_{t=1}^{T} CH_t$.

- $M_t = O_t + CH_t$:The total number of vessels (both company-owned and vessels available for chartering) of type t.
- $n = 1, ..., M_t$:Indices for vessels of type t.
- $n = 1, ..., O_t$:Indices for the company-owned vessels of type t.
- $n = O_t + 1, ..., O_t + CH_t \equiv M_t$:Indices for vessels of type t that are available for chartering.
- DC_t :Daily operational cost for a vessel of type t.
- $C_t = (DC_t) T_t$: Operational cost of a vessel of type t for making the round trip journey: source $\rightarrow$ destination $\rightarrow$ source.
- $\$_{t,n}$: Cost (in US dollars) of chartering a vessel n of type t.
- D :Total demand of the product over the duration of the time horizon.
- $100v$:Allowable percentage variation in the total demand.
- N :Number of demand partitions of the entire demand D.
- D_i :The i^{th} subdemand, for $i = 1, ..., N$.
- f_i and F_i: Respectively, denote the minimum and maximum allowable levels of the i^{th} subdemand D_i, for $i = 1, ..., N$.
- H_i : Entire delivery time interval associated with the i^{th} subdemand D_i, for $i = 1, ..., N$, where $H = \bigcup_{i=1}^{N} H_i$ and $H_i \cap H_j = \phi$, whenever $i \neq j$.
- $d_{1,i}$ and $d_{2,i}$: Respectively, denote the earliest and latest permitted penalty-free delivery days in H_i, for $i = 1, ..., N$.
- $H_i^{PF} = \{d_{1,i}, \ldots, d_{2,i}\}$, for $i = 1, ..., N$: The i^{th} penalty interval.
- $H^{PF} = \bigcup_{i=1}^{N} H_i^{PF}$: Entire penalty-free time-window.
- $\alpha_{1,i}$ and $\alpha_{2,i}$: Maximum days within which a shipment can be delivered before and after $d_{1,i}$ and $d_{2,i}$, respectively, but with some specified penalty as discussed later in Section 2.2, for $i = 1, ..., N$.
- $a_{1,i} = d_{1,i} - \alpha_{1,i}$ and $a_{2,i} = d_{2,i} + \alpha_{2,i}$, for $i = 1, ..., N$.
- $H_i^{PI} = \{a_{1,i}, ..., (d_{1,i} - 1)\} \cup \{(d_{2,i}$
- $+1), ..., a_{2,i}\}$: Type I penalty delivery time interval associated with the i^{th} subdemand D_i, for $i = 1, ..., N$.
- $H^{PI} = \bigcup_{i=1}^{N} H_i^{PI}$: The entire Type I delivery time window.
- $\beta_0 = 0$, $\beta_N = L$.
- $\beta_i = \frac{(a_{2,i} + a_{1,i+1})}{2}$, for $i = 1, ..., (N - 1)$.
- $H_{1,i}^{PII} = \{(\beta_{(i-1)} + 1), ..., (a_{1,i} - 1)\}$, for $i = 1, ..., N$.
- $H_{2,i}^{PII} = \{(a_{2,i} + 1), ..., \beta_i\}$, for $i = 1, ..., N$.
- $H_i^{PII} = H_{1,i}^{PII} \cup H_{2,i}^{PII}$, for $i = 1, ..., N$.
- $H^{PII} = \bigcup_{i=1}^{N} H_i^{PII}$:The entire Type II penalty delivery time window.

- $H_i = H_i^{PF} \cup H_i^{PI} \cup H_i^{PII}$ for $i = 1, ..., N$.
- $H = H^{PF} \cup H^{PI} \cup H^{PII}$.
- σ_i :Per barrel penalty associated with the i^{th} subdemand D_i for violation of the delivery time interval H_i^{PF}.

Example 1

The following is an illustrative example of the no penalty, Type I penalty, and Type II penalty time intervals.

- Let $\alpha_{1,i} = \alpha_{2,i} = 5,$ for $i = 1, ..., 3,$
- $L = 135,\ H = \{1, ..., 35\},\ N = 3,$
- $H_1 = \{1, ..., 45\}, H_2 = \{46, ..., 90\},$ $H_3 = \{91, ..., 135\},$
- $H_1^{PF} = \{15, ..., 30\}, H_2^{PF} = \{60, ..., 75\},$ $H_3^{PF} = \{105, ..., 120\},$
- $H_1^{PI} = \{10, ..., 14, 31, ..., 35\},$ $H_2^{PI} = \{55, ..., 59, 76, ..., 80\},$ $H_3^{PI} = \{100, ..., 104, 121, ..., 125\},$
- $H_1^{PII} = \{1, ..., 9, 36, ..., 45\},$ $H_2^{PII} = \{46, ..., 54, 81, ..., 90\},$ $H_3^{PII} = \{91, ..., 99\ ,\ 126, ..., 135\}.$
- $a_{1,1} = 10,\ a_{2,1} = 35,\ a_{1,2} = 55,$
- $a_{2,2} = 80,\ a_{1,3} = 100,\ a_{2,3} = 125,$
- $\beta_1 = 45,$ and $\beta_2 = 90.$

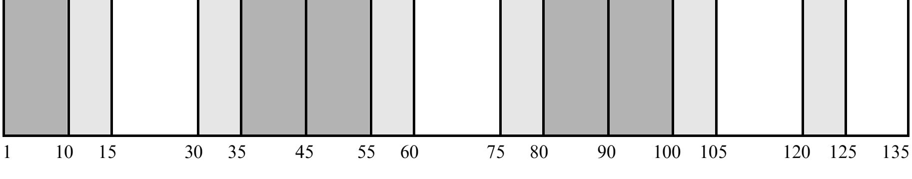
Then the no penalty (NP), Type I penalty (PI), and Type II penalty (PII) intervals are depicted in the time line as given by Figure 1.

Hence, the delivery of a shipment of the product is allowed on any day of the time horizon, either with no penalty associated with it or with a Type I or Type II penalty as illustrated in Figure 1.

A penalty function

A day $h \in H$ is a feasible starting day for a shipment carried by a vessel of type t with respect to the i^{th} subdemand D_i if $(h + T_{1,t} - 1) \in H_i$, where $T_{1,t}$ is as defined above. Hence, the shipload carried by this vessel is penalty-free if $(h + T_{1,t} - 1) \in H_i^{PF}$. On the other hand, if $(h + T_{1,t} - 1) \notin H_i^{PF}$, then this shipload is penalized a fixed amount based on **a)** the number of early or late days before $d_{1,i}$ or after $d_{2,i}$, **b)** the number of barrels in the shipment (i.e., the capacity of a vessel of type t given by Ω_t), and **c)** whether $(h + T_{1,t} - 1) \in H_i^{PI}$ or $(h + T_{1,t} - 1) \in H_i^{PII}$. Thus, the Type I and Type II penalty associated with a shipment carried by vessel n of type t and leaving the source on day h are deter-

Figure 1
Delivery time intervals



mined based on the delivery dates as follows:

Type I penalty:

$P_I(h,t,n) = \Omega_t \sigma_i$ maximum $\{(d_{1,i} - (h + T_{1,t} - 1)),\ ((h + T_{1,t} - 1) - d_{2,i})\}$ if $(h + (T_{1,t} - 1)) \in H_i^{PI}$, and

Type II penalty:

$$P_{II}(h,t,n) = \{ \alpha_{1,i}\Omega_t \sigma_i + \lambda, (a_{1,i} - (h + T_{1,t} - 1)), \text{ if } (h + (T_{1,t} - 1)) \in H_{1,i}^{PII},\ \alpha_{2,i}\Omega_t \sigma_i + \lambda ((h + T_{1,t} - 1) - a_{2,i}), \text{ if } (h + (T_{1,t} - 1)) \in H_{2,i}^{PII}.$$

Where λ is a sufficiently large number indicating the undesirability of delivering shipments in the Type II delivery time-window.

Accordingly, we define the following combined penalty function:

$$P(h,t,n) = \begin{cases} 0 & \text{if } (h + (T_{1,t} - 1) \in H_i^{PF}, \\ P_I(h,t,n) & \text{if } (h + (T_{1,t} - 1)) \in H_i^{PI}, \\ P_{II}(h,t,n) & \text{if } (h + (T_{1,t} - 1)) \in H_i^{PII}. \end{cases}$$

A Mixed-Integer Programming Scheduling Transportation Model

A mixed-integer programming model for the problem described in the foregoing sections is formulated below. The problem variables and constraints are presented first, and then the objective function and the model are given.

Problem variables

Define the following sets of binary decision variables as below.

Let

$$X_{h,t,n} = \begin{cases} 1 & \text{if vessel } n \text{ of type } t \text{ is dispatched on day } h \text{ from the source to the destination } n, \\ 0 & \text{otherwise} \end{cases}$$

A vessel cannot be dispatched from the source on day h of the time horizon unless it is available at the source on this day. The following set of binary decision variables represents the availability of vessels at the source.

Let

$$X_{h,t,n} = \begin{cases} 1 & \text{if vessel } n \text{ of type } t \text{ is available at the source on day } h, \\ 0 & \text{otherwise} \end{cases}$$

The chartering decisions variables are defined as follows.

Let

$$Z_{t,n} = \begin{cases} 1 & \text{if vessel } n \in \{O_t + 1, ..., M_t\} \text{ of type } t \text{ is selected for chartering during the time horizon}, \\ 0 & \text{otherwise} \end{cases}$$

A vessel n of type t may not be available at the source on day h of the time horizon because this vessel may be scheduled for maintenance on that day, or this vessel is company-owned, although it is involved in a trip from a previous contract. Hence, whenever $Y_{h,t,n} = 0$, this implies that the corresponding variable $X_{h,t,n} = 0$. Let Φ_X and Φ_Y denote the index sets of X and Y variables, respectively, which are restricted to be zero, based on considerations similar to the above.

For $i = 1, ..., N$, let U_i be a continuous variable representing the amount of the product that will be shipped from the source to the destination within the i^{th} delivery time interval H_i, and let U denote the total amount of the product that will be shipped to the destination over the entire time horizon.

Problem constraints

Constraints of the problem are formulated as follows..

a) Demand requirements

$$(C_1)\ \sum_t \sum_n \sum_{\substack{h: \\ (h+T_{1,t}-1) \in H_i}} \Omega_t X_{h,t,n} = U_i, \text{for } i = 1, ..., N,$$

$$f_i \leq U_i \leq F_i, \text{ for } \quad i = 1, ..., N,$$

$$(C_2) \quad \sum_h \sum_t \sum_n \Omega_t X_{h,t,n} = U,$$

$$D(1-v) \leq U \leq D(1+v).$$

For $i \in \{1, ..., N\}$, Constraints (C_1) assures that the total amount of the product shipped to the destination in the ith delivery time interval H_i lies within $[f_i, F_i]$, whereas Constraint (C_2) guarantees that the total demand requirement over the entire time horizon is satisfied within a permitted shortage or excess percentage given by v.

b) Vessel availability

$$(C_3) \quad Y_{h,t,n} = Y_{h-1,t,n} - X_{h-1,t,n} + \sum_{\substack{h_1 < h: \\ (h_1 + T_t - 1) = h}} X_{h_1,t,n}, \quad \forall\, h \geq 2, t, n,$$

$$(C_4) \quad X_{h,t,n} \leq Y_{h,t,n}, \quad \forall\, h, t, n.$$

A vessel n of type t can be dispatched from the source to the destination on day h of the time horizon only if it is available at the source on that day. This vessel is available at the source on day h if either the vessel was available at the source on the previous day and it was not dispatched, or this vessel was not available there during the previous day but it arrived on the current day. On the other hand, this vessel is unavailable on day h at the source if it was available there on the previous day and it was dispatched on that day (assuming that $T_t > 1$ for all vessel-types t), or it was unavailable on the previous day and it did not arrive on the current day. Constraint (C_3) examines the availability of vessel n of type t at the source on day h by incorporating these cases, and then Constraint (C_4) permits dispatching of vessels conditioned upon this availability.

c) Vessel chartering

The following constraint examines if a vessel $n \in \{O_t + 1, ..., M_t\}$ of type t is selected for chartering or not.

$$(C_5) \quad Y_{h,t,n} \leq Z_{t,n}, \quad \forall\, h,t,n \in \{O_t + 1, ..., M_t\}.$$

Hence, a shipment could be carried by a vessel $n \in \{O_t + 1, ..., M_t\}$ of type t on a given day of the time horizon, only if this vessel is selected for chartering, i.e., $Z_{t,n} = 1$.

Objective function

The objective function is composed of the following terms:

a) The operational costs of selected legs for both the company-owned and chartered vessels, which are given by $\sum_h \sum_t \sum_n C_t X_{h,t,n}$. Note that we assume that the operational cost of a vessel of type t is independent of h and is the same for all vessels of this type.

b) The Type I and Type II penalty associated with infeasible shipments as discussed in the previous section, which are given by

$$\sum_h P(h,t,n) X_{h,t,n}.$$

c) The chartering expenses are given by $\sum_t \sum_{n=O_t+1}^{M_t} \$_{t,n} Z_{t,n}$.

Thus, the objective function terms presented in **a)**, **b)**, and **c)** together with the constraints yield the following model (VSM) for the prescribed vessel scheduling problem. (All indices are assumed to take on only their respective relevant values).

VSM:

Minimize

$$\sum_h \sum_t \sum_n C_t X_{h,t,n} + \sum_h P(h,t,n) X_{h,t,n} + \sum_t \sum_{n=O_t+1}^{M_t} \$_{t,n} Z_{t,n},$$

subject to $(C_1), ..., (C_5)$,

$$X_{h,t,n} \in \{0,1\}, \quad \forall\, h,t,n \notin \Phi_X,$$

$$Y_{h,t,n} \in \{0,1\}, \quad \forall\, h,t,n \notin \Phi_Y,$$

$$Z_{t,n} \in \{0,1\}, \forall\, t,n \in \{O_t + 1, ..., M_t\},$$

$$f_i \leq U_i \leq F_i \text{ for } i = 1, ..., N,$$

$$D(1 - v) \leq U \leq D(1 + v).$$

Remark 1. Note that Constraint (C_3) is formulated so that the integrality of the Y-variables is guaranteed once the integrality of the X-variables and the Y-variables corresponding to the first day of the time-horizon is enforced. This follows since $Y_{h,t,n}$ is defined recursively in terms of $Y_{h-1,t,n}$ and a subset of the X variables. The integrality of the Z-variables is also automatically guaranteed. This follows because if for a given vessel type t, a vessel $n \in \{O_t + 1, ..., M_t\}$ is not used at all within the time horizon, then the most attractive value for $Z_{t,n}$ in the third term of the objective function is zero. On the other hand, if this vessel is involved in any trip between the source and the destination, then Constraints (C_4) and (C_5) force the $Z_{t,n}$ variable to have a value of one.

Remark 2. Capacity restrictions and maintenance requirements

A production capacity, say given by Q, might restrict the maximum amount of the product that can be shipped to the destination on any day of the time horizon. This restriction may be imposed via the following constraint.

$$(C_6) \sum_{t}\sum_{n} \Omega_t X_{h,t,n} \quad \leq \quad Q, \quad \forall\, h.$$

Also, maintenance considerations can be incorporated in the model. For example, the total usage of a vessel n of type t over the duration of the time horizon, say denoted by $TU_{t,n}$, might be specified *a priori*. This restriction can be imposed via the following constraint.

$$(C_7) \sum_{h}\sum_{t} T_t\, X_{h,t,n} \quad \leq \quad TU_{t,n}, \quad \forall\, t, n.$$

Other forms of scheduled maintenance restrictions can be also accommodated in the model by setting certain X-variables to zero.

Computational Results

This section presents computational results related to solving Model VSM directly by the CPLEX Optimization Package (version 7.5). Computational tests are performed on a set of six test problem instances (given in the Appendix A) that represent various operational scenarios.

We now define the following notation that will be used in this section.

- RT: Run time in CPU seconds on a Pentium IV computer having 256 MB of RAM and using CPLEX-7.5, where all procedures have been coded in Java.

For $k = 1, ..., 6$, let

- I_k denote the kth test problem,
- $v_{I_k}(\overline{\text{VSM}})$ denote the objective function value of the linear relaxation of Model VSM based on test problem I_k .
- $v_{I_k}(\text{VSM})$ denote the objective function value of Model VSM based on test problem I_k .

Tables 1 and 2 present computational results related to solving Model VSM using CPLEX.

Table 1
Statistics Related to Solving Model VSM via CPLEX

Test Problem	Number of Rows	Number of Columns	Number of Nonzero Entries	$v_{I_k}(\overline{\text{VSM}})$ (\$)	RT
I_1	1,119	871	3,457	6,100,060	0.28
I_2	1,023	861	3,275	2,900,005	0.26
I_3	3,348	2,482	10,150	12,340,000	0.26
I_4	2,781	2,443	9,055	6,400,000	0.26
I_5	5,172	3,990	15,894	17,100,000	0.35
I_6	4,749	3,996	15,072	13,940,000	0.28

Table 2
Statistics Related to Solving Model VSM via CPLEX (continued)

Test Problem	v_{I_k} (VSM) ($)	RT
I_1	6,600,080	212.12
I_2	3,400,010	126.10
I_3	12,460,010	450.70
I_4	6,680,000	510.27
I_5	17,540,000	1009.50
I_6	14,280,000	1981.33

Comparison with an *ad-hoc* scheduling procedure

In this section, we compare the performance of Model VSM with an *ad-hoc* scheduling procedure that is intended to simulate how vessel schedules are generated manually. Note that this *ad-hoc* procedure, denoted by AH, is not unique and perhaps different organizations utilize different *ad-hoc* scheduling schemes. Nonetheless, due to the combinatorial nature of the problem, any *ad-hoc* procedure is likely to be inefficient and costly. Since chartering expenses are of large magnitudes relative to costs of legs and penalties imposed on early or late shipments, the procedure attempts to fully utilize company-owned vessels before resorting to chartered vessels. In general, this procedure is described in two steps as follows. For $i \in \{1, ..., N\}$, let SL_i denote the (so far) total amount of the product delivered to the destination to satisfy the i^{th} subdemand.

A) Utilization of the company-owned vessels

To avoid chartering of vessels, the procedure attempts to maximally utilize the company- owned vessels by minimizing the idle times of vessels during the time horizon. Hence, for a vessel n of type t, the procedure finds the smallest h that satisfies the following conditions:

a) This vessel is available for dispatching from the source on day h, i.e., this vessel is not used in any previously determined source-destination roundtrip that involves day h.

b) If $h + T_{1,t} - 1 \in H_i$ for some $i \in \{1, ..., N\}$, then $SL_i < f_i$.

c) $SL_i + \Omega_t \leq F_i$.

Now if conditions a, b and c are satisfied, then vessel n of type t is dispatched and the total cost is updated by adding the operational cost of this trip in addition to any Type I or Type II penalty.

B) Resorting to chartered vessels as needed

If $SL_i < f_i$ for some $i \in \{1, ..., N\}$, then the vessel having the least chartering cost is selected and the corresponding chartering cost is added to the total cost. The dispatching decisions of this vessel is determined in a similar fashion as discussed in A. Step B is then repeated until $SL_i \geq f_i$ for all $i \in \{1, ..., N\}$.

Table 3 compares the costs of schedules obtained via Model VSM and Procedure AH.

It can be observed that for each test problem, the overall transportation cost obtained via Model VSM is as good as, and often substantially better, than that obtained via Procedure AH. For example, for test problem I_4, the gap between the overall cost obtained via Model VSM and that obtained via Procedure AH is quite large (65.9%).

Summary, Concluding Remarks, and Future Research

This paper examines a storage replenishment ocean transportation problem faced by an oil company in delivering a product from a source to a destination based on specified delivery time intervals. A mixed-integer programming model is formulated for the problem. A solution of the model determines the fleet size mix being composed of company-owned and chartered vessels, the dispatching dates of vessels, and the penalties incurred on some shipments that are not delivered within any of the penalty-free time intervals. Due to the high cost of chartering, the model attempts to optimally utilize company-owned vessels, to the extent possible, and then resort to chartering as needed. The model is solved using the CPLEX (version 7.5) optimization package based on a number of test cases representing various operational scenarios. Schedules generated using a simple *ad-hoc* procedure are compared with those obtained via the proposed modeling approach. The results indicate that the costs of sche-

Table 3
Comparison between costs of schedules obtained via Model VSM and Procedure AH

Test Problem	v_{I_k} (VSM) ($)	AH ($)	$\left(\frac{\text{AH}-v_{I_k}\text{(VSM)}}{\text{AH}}\right)100$ (Percentage of Improvement of Model VSM over Procedure AH)
I_1	6,600,080	8,600,350	23.2%
I_2	3,400,010	8,600,350	60.4%
I_3	12,460,010	21,760,570	42.7%
I_4	6,680,000	19,600,650	65.9%
I_5	17,540,000	28,540,850	38.5%
I_6	14,280,000	27,200,950	47.5%

dules generated via the model are noticeably less than the costs of schedules obtained by the *ad-hoc* procedure. The potential saving and usefulness of the model in planning and negotiation purposes strongly justifies its use in practice. In particular, by running the model in sensitive analysis fashion for different possible delivery and penalty alternatives, the company can examine the impact of a given contract in its overall operation and net profits. Ongoing extensions of this work include the development of mathematical models for the following scenarios: **a)** multiple sources and destinations operations, **b)** the consideration of a demand structure that is determined based on daily consumptions of product(s) at destination(s) storage facilities, **c)** the impact of leasing transshipment depots both on the single source-destination and multiple sources-destinations operations, and **d)** incorporating stochastic aspects of the various demand structures and travel times into these models

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Appendix A
Test problems

This appendix presents statistics related to six test problems. Assume that there are three types of vessels: Type I Penalty is fixed at \$0.5 for all the subdemands, Type II Penalty is fixed at \$1 and all vessels of the same type are chartered for the same price that is denoted by $\$_t$.

Table A1
Vessel-types

t	$T_{1,t}$ (days)	Ω_t (barrels)	$\$_t$ (\$)	DC_t (\$)
1	5	400,000	\$2,000,000	\$20,000
2	8	600,000	\$2,200,000	\$30,000
3	11	800,000	\$2,400,000	\$40,000

For a given nonnegative integer i, Table A2 gives the various penalty intervals associated with subdemand D_i.

Table A2
Penalty and no-penalty time intervals for subdemand D_i

Interval (days)	Penalty Type
$\{45(i-1)+1, ..., 45(i-1)+9\}$	Type II
$\{45(i-1)+10, ..., 45(i-1)+14\}$	Type I
$\{45(i-1)+15, ..., 45(i-1)+30\}$	No penalty
$\{45(i-1)+31, ..., 45(i-1)+35\}$	Type I
$\{45(i-1)+36, ..., 45(i-1)+45\}$	Type II

Note that for $i = 1,2$, and 3, the intervals determined via Table A2 coincide with those given in Example 1. It is worth mentioning that the structure of the intervals given in Table A2 is basically the same for all subdemands D_i in the sense that the duration of Type II, Type I, and the no penalty intervals is the same for all such subdemands. Nonetheless, the modeling approach presented in this paper can easily accommodate other structures of penalty intervals. Table A3 defines upper and lower quantities associated with D_i for $i = 1, 2, 3$.

Table A3
Lower and upper quantities associated with the subdemand D_i

i	$[f_i, F_i]$ (barrels)
1	[5,000,000 - 6,000,000]
2	[6,000,000 - 7,000,000]
3	[7,000,000 - 8,000,000]

Table A4
Test Problems

Test Problem I_k	H (days)	N (vessel types)	(T, O, CH) (vessels)	(t, O_t, CH_t) (vessels)
I_1	$\{1, \ldots, 45\}$	1	(1,4,6)	(1,4,6)
I_2	$\{1, \ldots, 45\}$	1	(1,6,4)	(1,6,4)
I_3	$\{1, \ldots, 90\}$	2	(2,4,10)	(1,2,5) , (2,2,5)
I_4	$\{1, \ldots, 90\}$	2	(2,10,4)	(1,5,2) , (2,5,2)
I_5	$\{1, \ldots, 135\}$	3	(3,6,9)	(1,2,3) , (2,2,3) , (3,2,3)
I_6	$\{1, \ldots, 135\}$	3	(3,9,6)	(1,3,2) , (2,3,2) , (3,3,2)

Appendix B

This appendix presents detailed schedules obtained via Model VSM and the *ad-hoc* procedure AH. The schedules are prescribed via the triples (h, t, n), where (h, t, n) indicates that vessel n of type t will be dispatched form the source to the destination on day h. Note that if $n \in \{1, ..., O_t\}$, then the vessel is a self-owned one, while if $n \in \{O_t + 1, ..., M_t\}$, then the vessel is a chartered one.

Table B1
Vessel schedules generated via Model VSM

Test Problem	Trips (h, t, n)	Operational Cost ($)	Penalty Cost ($)	Chartering Cost ($)	Total Cost ($)
I_1	(5,1,1), (15,1,1), (25,1,1), (2,1,2), (12,1,2), (22,1,2), (32,1,2), (5,1,3), (15,1,3), (26,1,3), (5,1,4), (16,1,4), (26,1,4)	2,600,000	4,000,080	0	6,600,080
I_2	(13,1,1), (23,1,1), (11,1,2), (21,1,2), (11,1,3), (21,1,3), (32,1,3), (11,1,4), (21,1,4), (11,1,5), (23,1,5), (11,1,6), (25,1,6)	2,600,000	800,010	0	3,400,010
I_3	(12,1,1), (25,1,1), (58,1,1), (71,1,1), (16,1,2), (26,1,2), (56,1,2), (66,1,2), (12,1,4), (23,1,4), (56,1,4), (66,1,4), (77,1,4), (11,1,5), (21,1,5), (7,2,1), (23,2,1), (52,2,1), (68,2,1), (17,2,2), (52,2,2), (68,2,2)	6,760,000	1,700,010	4,000,000	12,460,010
I_4	(11,1,1), (21,1,1), (56,1,1), (68,1,1), (15,1,2), (25,1,2), (66,1,2), (11,1,3), (22,1,3), (56,1,3), (66,1,3), (11,1,4), (21,1,4), (56,1,4), (66,1,4), (11,1,5), (21,1,5), (56,1,5), (66,1,5), (68,2,1), (17,2,3), (68,2,3), (68,2,4), (17,2,5), (53,2,5)	6,680,000	0	0	6,680,000
I_5	(11,1,1), (22,1,1), (58,1,1), (68,1,1), (105,1,1),(115,1,1), (11,1,2), (21,1,2), (66,1,2), (101,1,2), (112,1,2),(11,1,4), (24,1,4), (71,1,4), (101,1,4), (112,1,4), (11,1,5), (21,1,5), (56,1,5), (67,1,5), (101,1,5), (112,1,5), (17,2,1), (68,2,1), (97,2,1), (113,2,1), (7,2,2), (23,2,2), (54,2,2), (98,2,2), (114,2,2), (50,3,1), (95,3,1), (50,3,2), (95,3,2)	12,640,000	900,000	4,000,000	17,540,000
I_6	(11,1,1), (22,1,1), (56,1,1), (68,1,1), (104,1,1),(114,1,1), (11,1,2), (21,1,2), (56,1,2), (67,1,2), (105,1,2),(115,1,2), (11,1,3), (21,1,3), (56,1,3), (66,1,3), (101,1,3),(111,1,3), (17,2,1), (65,2,1), (97,2,1), (113,2,1), (17,2,2), (97,2,2), (113,2,2), (17,2,3), (68,2,3), (113,2,3), (17,3,1), (50,3,1), (50,3,2), (95,3,2), (50,3,3), (95,3,3)	13,680,000	600,000	0	14,280,000

Table B2
Vessel schedules generated via the *ad-hoc* procedure AH

Test Problem	Trips (h, t, n)	Operational Cost ($)	Penalty Cost ($)	Chartering Cost ($)	Total Cost ($)
I_1	(1,1,1), (11,1,1), (21,1,1), (31,1,1), (41,1,1), (1,1,2), (11,1,2), (21,1,2), (31,1,2), (41,1,2), (1,2,1), (11,2,1), (21,2,1)	2,600,000	6,000,350	0	8,600,350
I_2	(1,1,1), (11,1,1), (21,1,1), (31,1,1), (41,1,1), (1,1,2), (11,1,2), (21,1,2), (31,1,2), (41,1,2), (1,2,1), (11,2,1), (21,2,1)	2,600,000	6,000,350	0	8,600,350
I_3	(1,1,1), (11,1,1), (21,1,1), (31,1,1), (41,1,1), (51,1,1), (61,1,1), (71,1,1), (81,1,1), (1,1,2), (11,1,2), (21,1,2), (31,1,2), (41,1,2), (51,1,2), (61,1,2), (71,1,2), (81,1,2), (1,2,1), (17,2,1), (33,2,1), (49,2,1), (65,2,1), (82,2,1), (1,2,2)	6,960,000	14,800,570	0	21,760,570
I_4	(1,1,1), (11,1,1), (21,1,1), (31,1,1), (41,1,1), (51,1,1), (61,1,1), (71,1,1), (81,1,1), (1,1,2), (11,1,2), (21,1,2), (31,1,2), (41,1,2), (51,1,2), (61,1,2), (71,1,2), (81,1,2), (1,1,3), (11,1,3), (21,1,3), (31,1,3), (41,1,3), (51,1,3), (61,1,3), (71,1,3), (81,1,3), (1,1,4),	5,600,000	14,000,650	0	19,600,650
I_5	(1,1,1), (11,1,1), (21,1,1), (31,1,1), (41,1,1), (51,1,1), (61,1,1), (71,1,1), (81,1,1), (91,1,1), (101,1,1),(111,1,1), (121,1,1), (131,1,1), (1,1,2), (11,1,2), (21,1,2), (31,1,2), (41,1,2), (51,1,2), (61,1,2), (71,1,2), (81,1,2), (91,1,2), (101,1,2),(111,1,2),(121,1,2), (131,1,2), (1,2,1), (17,2,1), (33,2,1), (49,2,1), (65,2,1), (82,2,1), (97,2,1), (112,2,1)	9,440,000	19,100,850	0	28,540,850
I_6	(1,1,1), (11,1,1), (21,1,1), (31,1,1), (41,1,1), (51,1,1), (61,1,1), (71,1,1), (81,1,1), (91,1,1), (101,1,1), (111,1,1), (121,1,1), (131,1,1), (1,1,2), (11,1,2), (21,1,2), (31,1,2), (41,1,2), (51,1,2), (61,1,2), (71,1,2), (81,1,2), (91,1,2), (101,1,2),(111,1,2),(121,1,2), (131,1,2), (1,1,3), (11,1,3), (21,1,3), (31,1,3), (41,1,3), (51,1,3), (61,1,3), (71,1,3), (81,1,3), (91,1,3), (101,1,3), (111,1,3)	8,000,000	19,200,950	0	27,200,950

الملخص

نموذج برمجة رياضية لمشكلة النقل البحري

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جامعة الكويت

يعنى هذا البحث بتحديد أفضل مزيج من السفن المملوكة والمؤجرة لنقل البضائع ما بين نقطتين (المصدر والمكان المقصود). ومن الصفات التي تؤخذ بعين الاعتبار لنوع الناقلات المستخدمة السعة، والتكاليف التشغيلية اليومية، والسرعة وتكاليف التأجير. ومن الفوائد المترتبة من التخطيط الكفء لجدولة الناقلات التوفير الملحوظ من ناحية التكلفة، وبخاصة التكاليف الباهظة المترتبة على إدارة عمليات النقل للسفن المملوكة والسفن المؤجرة، وكذلك التكاليف المترتبة على العقوبات التي تفرض بسبب انتهاك شروط التوريد. وفي هذا البحث سوف يتم تطوير نموذج باستخدام برمجة الأعداد الصحيحة Integer-Programming لتحديد الحجم المناسب لأسطول الناقلات ما بين الممتلك منه والمؤجر، وكذلك سوف يعرض البحث الحلول للنماذج المقترحة التي تصف الرحلة، وتواريخ التوصيل للناقلات المملوكة والناقلات المختارة للتأجير بالنسبة لكل نوع من أنواع السفن، وكذلك للعقوبات اليومية الناتجة عن المخالفات لشروط التوصيل بسبب التكلفة العالية الناشئة عن تأجير الناقلات، والنموذج المقترح يهدف إلى استخدام الناقلات المملوكة بالطريقة المثلى، والتقليل من استخدام الناقلات المؤجرة. وسوف يجري حل النماذج المقترحة باستخدام برنامج CPLEX-7.5 لمجموعه من الأمثلة، كما سيجرى رصد هذه النماذج وتحليلها.

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