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A MINIMAL SPANNING TREE MODEL FOR WATER ALLOCATION

Key Words

Network; Minimal Spanning Tree; Gravity; Pumping; Optimal Solution.

Abstract

The allocation and distribution of water to Southern Lebanese towns, which have long suffered from lacking in water distribution and scarcity, is among key reconstruction and rehabilitation projects that consume a part of the public budget of the region. Many factors influence the planning of water distribution to towns: the capacities, the needs and the cost of equipment of installation. The authorities in the region aim to provide water to 29 southern towns (via a principal water allocation network). It is required to determine the links that will result in the use of minimum water pipes while guaranteeing that all the areas will be connected (directly or indirectly) to the main water station. We used a minimal spanning tree technique to address this question. The importance of this approach lies in its ability to provide an optimal solution that allows the execution of this project at minimum costs. This study results in costs that are much lower than the costs of an existing study that doesn't take into account the scientific method to find the best solution.

Introduction

The municipal water managers (MWM) are in the process of planning a network for providing water to 29 towns, situated in South Lebanon. These towns, which have a population of 100 000 people, will profit from this water allocation for 22 years.

The main station that will provide water to all these towns is located in a town called Tefahta. It is at about 345 meters above the sea. Wells were dug in this area for this purpose. In each town a station will be installed and

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nourished by the main station in order to provide water to the town.

Many factors affect the allocation of water in the region. These include the geographical location of each town with respect to the main station, the amount of water needed in each town, the cost of the installation of the various equipment and the capacities of providing water.

The towns involved in this project are distributed in a manner where some of them are located in areas higher than the main station (the highest one is at 400 meters above the sea) and the others are situated in areas under Tefahta (the lowest one is at the sea level).

In order to minimize the costs of installation, it was decided to connect the towns to the main station by installing equipment under the paved roads that connect all these towns.

The municipal water managers (MWM) were concerned with the costs of the project and needed to determine under which roads the water pipes should be installed to connect all the towns with a minimum total length of pipes.

In this study, we applied the minimal spanning tree (MST) model to minimize the total distance of the network that will provide the 29 towns with water.

Minmal Spanning Tree for Water Allocation

The minimal spanning tree algorithm seeks to connect all the points (nodes) of a network together while minimizing the distance between them (Render and Stair, 1994). When the points represent towns in a region, the minimal-spanning tree technique can be used to determine the best way to connect all the towns to the main water station, in a way that minimizes the total distance or length of water pipes.

Because the installation is expensive, pipes will be installed under just enough roads to provide connection between the towns and the main station.

The question is where the pipes should be laid to accomplish this minimum total number of pipes installed. Because of the nature of this problem, we used a minimal spanning tree to find an optimal solution.

The minimal-spanning tree technique that will be used in this study is outlined below [Render and Stair, 1994]:

1. Select any node in the network.
2. Connect this node to the nearest node that minimizes the total distance.
3. Considering all of the nodes that are now connected, find and connect the nearest node that is not connected.

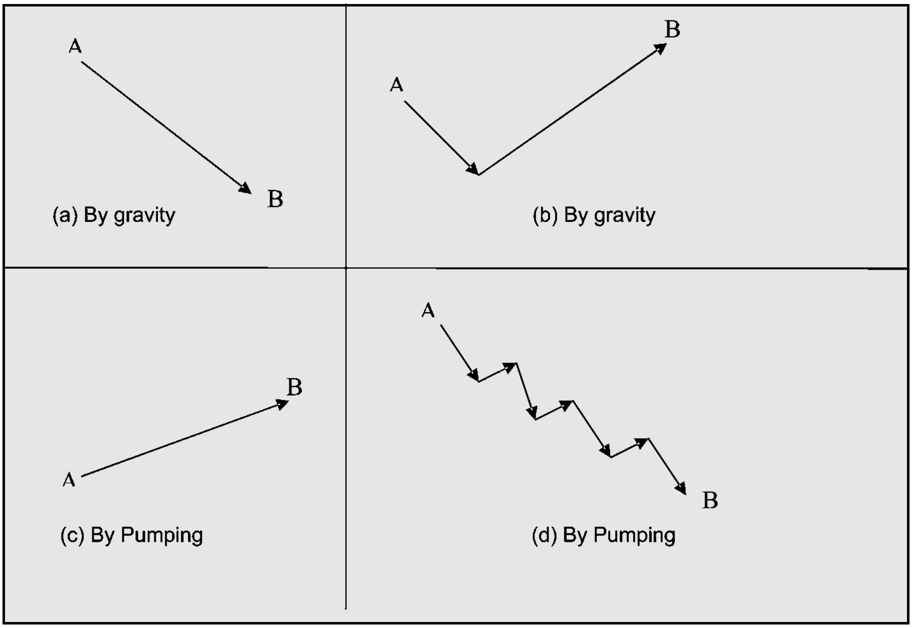
4. Repeat the third step until all nodes are connected.

5. If there is a tie in the third step and two or more nodes that are not connected and are equally near, arbitrarily select one and continue. A tie suggests that there might be more than one optimal solution.

Details of Water Management

The geographical pattern of the region under study makes some towns higher than the main station in Tefahta and other ones lower. The geographical relief between each pair of towns A and B was classified into one of the four categories shown in figure (1).

Figure 1
The Different Geographical Relieves Between A Pair of Towns A and B and the Way they are Provided with Water



As you notice in (a) the way between A and B is gradually declined, so water can flow by force of gravity between these two towns. Whereas in (b) water falls strongly to a certain level so that it is pushed up to a higher place without the use of any pumps. This pattern was classified with towns nourished by water by gravity.

On the other hand, we can see in (c) another form of ways that goes up gradually so that water needs pumps to flow from A to B. Although in (d) the way between towns A and B goes downwards, but it is full of ups and downs, so we need pumps to push water to flow from A to B.

These classifications were put by engineers specialized in this field. Based on the new distribution of the towns we obtained 2 sub-networks: the first one contains the towns that can be provided with water by pumping and the second contains the towns that can be nourished by the force of gravity.

The distances between towns, the altitudes in meters above the sea, and the water flow are given in table (1). The altitude of each town is, in general, important to determine the order in which towns will be nourished by gravity. The gravity network is constructed in a way that it passes from one town to another with less altitude, except some cases that are shown in figure (1-b).

Table 1
Towns Included in the Study with Distances Between them, their Altitudes, and Water Flow Possibilities. (G Means Gravity and P Pumping).

Towns	Distance	Height		Flow	Remarks
Town A ˜ Town B		A	B		
Tefahta ˜ Kawtharyet-Al Syad	4700	345	300	P-G	2 lines
Tefahta ˜ Kaakaiet-Al-Sanawbara	6945	345	240	G	
	3845	345	240	G	
Tefahta ˜ Al-babilieh	4825	345	220	G	
Tefahta ˜ Al-bissarieh	5590	345	180	G	
Tefahta ˜ Coast line	10700	345	21	P	
Kawtharyet-Al Syad ˜ Al-Khartum	1950	300	300	P	
Kawtharyet-Al Syad ˜Kaakaiet-Al-Sanawbara	4230	300	240	G	
Kawtharyet-Al Syad ˜ Al Ghassanieh	3300	300	330	P	
Kawtharyet-Al Syad ˜ Al-babilieh	4350	300	220	G	
Kawtharyet-Al Syad ˜ Al-Doueir	6200	300	370	P	
Kawtharyet-Al Syad ˜ Al-Sharkieh	3670	300	400	P	
Kawtharyet-Al Syad ˜ Shel-Baal	8070	300	360	P	
Kakaiet-Al-Sanawbara ˜ Al-Bissarieh	2940	240	180	G	
	2920	240	180	G	
	2470	240	180	G	
Kakaiet-Al-Sanawbara ˜ Al-Babilieh	2180	240	220	G	
Kakaiet-Al-Sanawbara ˜ Al-Ghassanieh	3130	240	330	P	
Kakaiet-Al-Sanawbara ˜ Coast Line	6140	240	21	G	
Al-Bissarieh ˜ Al-Babilieh	4630	180	220	G	
	3865	180	220	G	
Al-Bissarieh ˜ Kherbet-Al-Doueir	2220	180	200	G	
Al-Bissarieh ˜ Al-Akbieh	2620	180	47	G	
Al-Bissarieh ˜ Al-Sarafand	2640	180	155	G	
Al-Babilieh ˜ Kherbet-Al-Doueir	4600	220	200	G	
Al-Babilieh ˜ Ansarieh	7050	220	170	G	
Al-Babilieh ˜ Al-Dawoudieh	4760	220	165	G	
Kherbet-Al-Doueir ˜ Al-Babilieh	4600	200	138	G	
Kherbet-Al-Doueir ˜ Al-Dawoudieh	1700	200	165	G	
Dayet-Al-Arab ˜ Al-Akbieh	1990	138	47	G	
Al-Dawoudieh ˜ Al-Saksakieh	3000	165	100	G	
Al-Dawoudieh ˜ Ansarieh	3900	165	170	G	
Al-Dawoudieh ˜ Al-Sarafand	2700	165	70	G	

Cont. Table 1
Towns Included in the Study with Distances Between them, their Altitudes, and Water Flow Possibilities. (G Means Gravity and P Pumping).

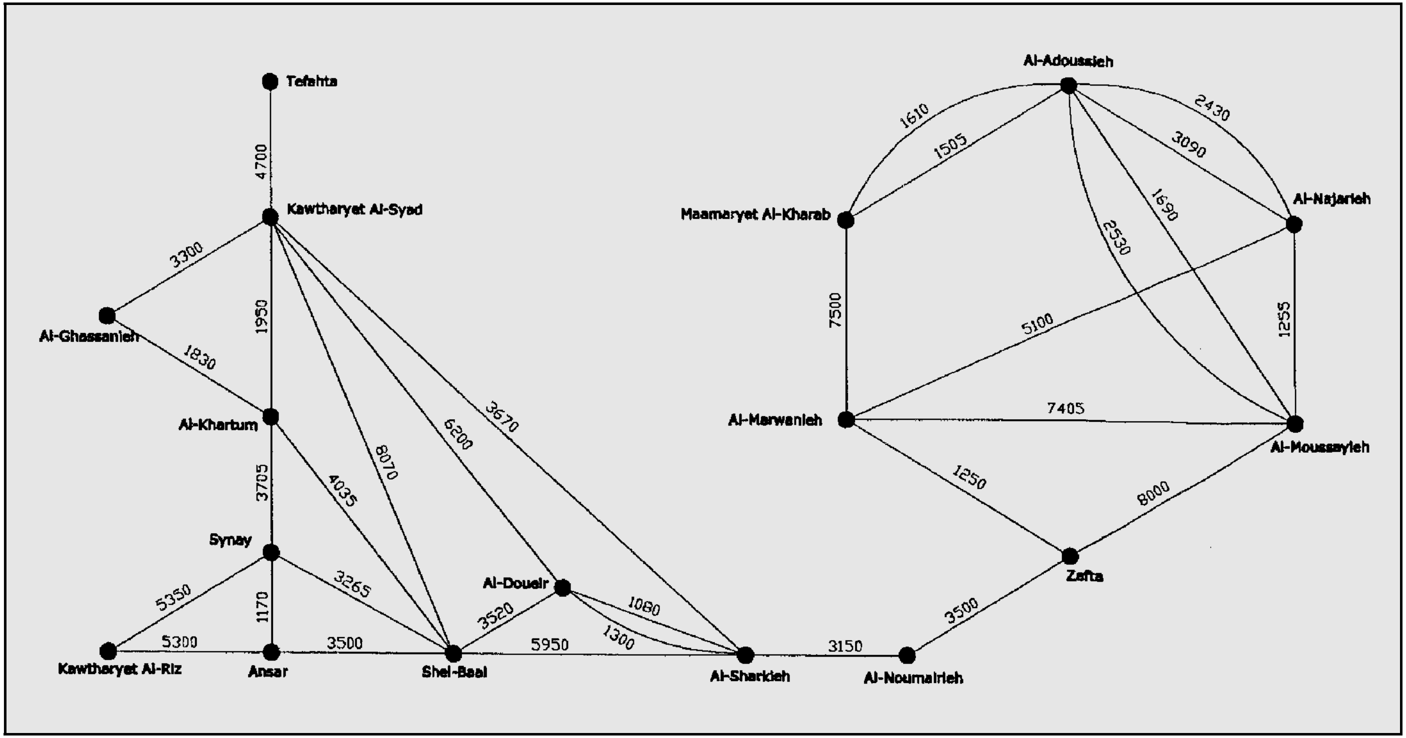
Towns	Distance	Height		Flow	Remarks
Town A ˜ Town B		A	B		
Al-Sarafand ˜ Dayet-Al-Arab	500	70	138	G	
Al-Sarafand ˜ Al-Saksakieh	2800	70	100	G	
Al-Akbieh ˜ Coast Line	1950	47	21	G	
Ansarieh ˜ Adloun	900	170	155	G	Needs digging + Pipe cost.
Ansarieh ˜ Al-Loubieh	2100	170	154	G	Needs pipe cost only.
Al-Sharkieh ˜ Al-Doueir	1080	400	370	P	
	1300	400	370	P	
Al-Sharkieh ˜ Al-Noumayrieh	3150	400	323	P	
Al-Sharkieh ˜ Shel-Baal	5950	400	360	P	
Al-Noumayrieh ˜ Zefta	3500	323	340	P	
Al-Marwanieh ˜ Meamaryat-Al-kharab	7500	340	250	P	
Al-Marwanieh ˜ Al-Moussayleh	7405	340	125	P	
Al-Marwanieh ˜ Al-Najarieh	5100	340	140	P	
Al-Marwanieh ˜ Zefta	1250	340	340	P	
Maamouryet El Kharab ˜ Al Adoussieh	1505	250	100	P	
Al Adoussieh ˜ Al-Najarieh	3090	100	140	P	
	2430	100	140	P	
Al Adoussieh ˜ Al-Moussayleh	1690	100	125	P	
	2530	100	125	P	
Zefta ˜ Al-Moussayleh	8000	340	125	P	
Al-Najarieh ˜ Al-Moussayleh	1255	140	125	P	
Al-Ghassanieh ˜ Al-Khartum	1830	330	300	P	
Al-Khartum ˜ Synay	3705	300	320	P	
Al-Khartum ˜ Shel-Baal	4035	300	360	P	
Synay ˜ Ansar	1170	320	300	P	
Al-Doueir ˜ Shel-Baal	3520	370	360	P	
Shel-Baal ˜ Synay	3265	360	320	P	
Shel-Baal ˜ Ansar	3500	360	300	P	
Kawtharyet-Al Riz ˜ Synay	5350	180	320	P	
Kawtharyet-Al Riz ˜ Ansar	5300	180	300	P	
Coast Line ˜ Al-Adoussieh	2525	21	100	P	
Coast Line ˜ Al-Najarieh	2250	21	140	P	
Coast Line ˜ Al-Moussayleh	1615	21	125	P	
Coast Line ˜ Al-Akbieh	1950	21	47	P	

Sources: all numbers used in this table are taken from the Ministry of public works and the Ministry of Defense.

A graphical depiction of the problem shows all the possible connections between towns in each line: pumping line (figure 2) and line gravity (figure 3).

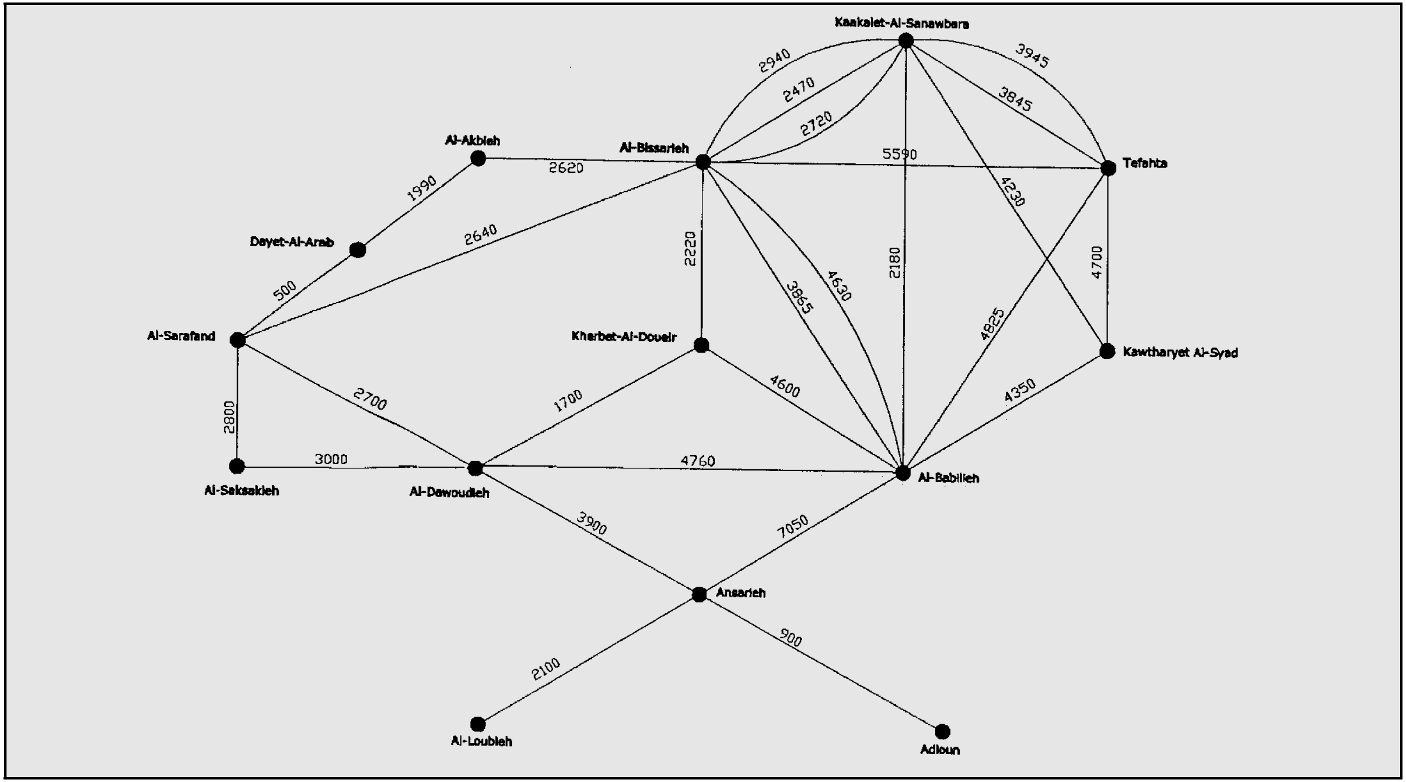
A branch connecting any pair of towns indicates the presence of a paved road between them and the number associated to a branch represents the distance between the linked towns. Some towns do not have branches between them; this means that there are no direct paved roads between them.

Figure 2
Graphical Representation of all Possible Connections Between Towns in the Pumping Line



Sourse: Distances between towns are taken from the ministry of public Works (all numbers are in meters).

Figure 3
Graphical Representation of all Possible Connections Between Towns in the Gravity Line



Sourse: Distances between towns are taken from the ministry of public Works (all numbers are in meters).

Results

To solve the minimal spanning tree (MST) problem and find the optimal network for both cases (water allocation by gravity and water allocation by pumping) we used Tora software and obtained the following results:

Table 2
Towns Included in the Optimal Network for Water Allocation by Gravity and Distances in Meters

Towns	Distance
Town A ˜ Town B	
Tefahta ˜ Kaakaiet-Al-Sanawbara	3845
Kaakaiet-Al-Sanawbara ˜ Kawtharyet Al-Syad	4230
Kaakaiet-Al-Sanawbara ˜ Al-Babilieh	2180
Kakaiet-Al-Sanawbara ˜ Al-bissarieh	2470
Kakaiet-Al-Sanawbara ˜ Al-bissarieh	2220
Kherbet-Al-Doueir ˜ Al-Dawoudieh	1700
Al-Dawoudieh ˜ Ansarieh	3900
Ansarieh ˜ Adloun	900
Ansarieh ˜ Al-Loubieh	2100
Al-bissarieh ˜ Al-Akbieh	2620
Al-Akbieh ˜ Dayet-Al-Arab	1990
Dayet-Al-Arab ˜ Al-Sarafand	500
Al-Sarafand ˜ Al-Sharkieh	2800
Total	31455

Table 3
Towns Included in the Optimal Network for Water Allocation by Pumping and Distances in Meters

Towns	Distance
Town A ˜ Town B	
Tefahta ˜ Kawtharyet-Al Syad	4700
Kawtharyet-Al Syad ˜ Al-Khartum	1950
Al-Khartum ˜ Al-Ghassanieh	1830
Kawtharyet Al-Syad ˜ Al Sharkieh	3670
Al-Sharkieh ˜ Al-Doueir	1080
Al-Doueir ˜ Shel-Baal	3520
Shel-Baal ˜ Synay	3265
Synay ˜ Ansarieh	1170
Ansar ˜ Kawtharyat-Al Riz	5300
Al-Sharkieh ˜ Al-Noumayrieh	3150
Al-Noumayrieh ˜ Zefta	3500
Zefta ˜ Al-Marwanieh	1250
Al-Marwanieh ˜ Al-Najarieh	5100
Al-Najarieh ˜ Al-Moussayleh	1255
Al-Moussayleh ˜ Al-Adoussieh	1690
Al-Adoussieh ˜ Meamaryat-Al-kharab	1505
Total	43935

When Water Is Provided By Gravity

We solved the MST model for the condition when water is provided by gravity. The solution obtained (table 2) indicated that the optimal network length is 31455 meters, which is lower than the existing study network that required 40535 meters of water pipes (see table 4).

When Water Is Allocated By Pumping

We modeled that situation when water is allocated by pumping. The optimal solution (table 3) indicates that the optimal network length is 43935 meters, which is lower than the existing study network that needs 65495 meters of water pipes (see table 5).

Table 4
Towns Included in the Existing Network Study for Water Allocation by Gravity and Distances en Meters

Towns	Distance
Town A ˜ Town B	
Tefahta ˜ Kawtharyat-Al Syad	4700
Kawtharyat Al-Syad ˜ Kaakaiet-Al-Sanawbara	3845
Kawtharyat Al-Syad ˜ Al-Babilieh	4350
Kakaiet-Al-Sanawbara ˜ Al-bissarieh	2470
Al-bissarieh ˜ Al-Akbieh	2620
Al-Babilieh ˜ Kherbet-Al-Doueir	4600
Al-Babilieh ˜ Ansar	7050
Kherbet-Al-Doueir ˜ Al-Sarafand	2700
Al-Sarafand ˜ Dayet-Al-Arab	500
Ansarieh ˜ Al-Loubieh	2100
Ansarieh ˜ Adloun	900
Kherbet-Al-Doueir ˜ Al-Dawoudieh	1700
Al-Dawoudieh ˜ Al-Sharkieh	3000
Total 31455	40535

Table 5
Towns Included in the Existing Network for Water Allocation by Pumping and Distances in Meters

Towns	Distance
Town A ˜ Town B	
Tefahta ˜ Kawtharyat-Al Syad	4700
Kawtharyat-Al Syad ˜ Al-Ghassanieh	3300
Kawtharyat-Al Syad ˜ Al-Khartum	1950
Kawtharyat Al-Syad ˜ Al-Doueir	6200
Al-Doueir ˜ Shel-Baal	8070
Al-Doueir ˜ Al-Sharkieh	1300
Shel-Baal ˜ Ansar	3500
Ansar ˜ Synay	1170
Ansar ˜ Kawtharyet-Al Riz	5300
Al-Sharkieh ˜ Al-Noumayrieh	3150
Al-Noumayrieh ˜ Zefta	3500
Zefta ˜ Al-Moussayleh	8000
Zefta ˜ Al-Marwanieh	1250
Al-Marwanieh ˜ Al-Najarieh	5100
Al-Marwanieh ˜ Meamaryat-Al-kharab	7500
Meamaryat-Al-kharab ˜ Al-Adoussieh	1505
Total	65490

A graphical representation of the two optimal networks (MST solutions for water allocation by gravity and water allocation by pumping) is given in figures 4 and 5.

In each network we can see the order in which the towns are provided by water.

Finally, we compared the existing water allocation study and the MST solutions for the two-modeled conditions we find that:

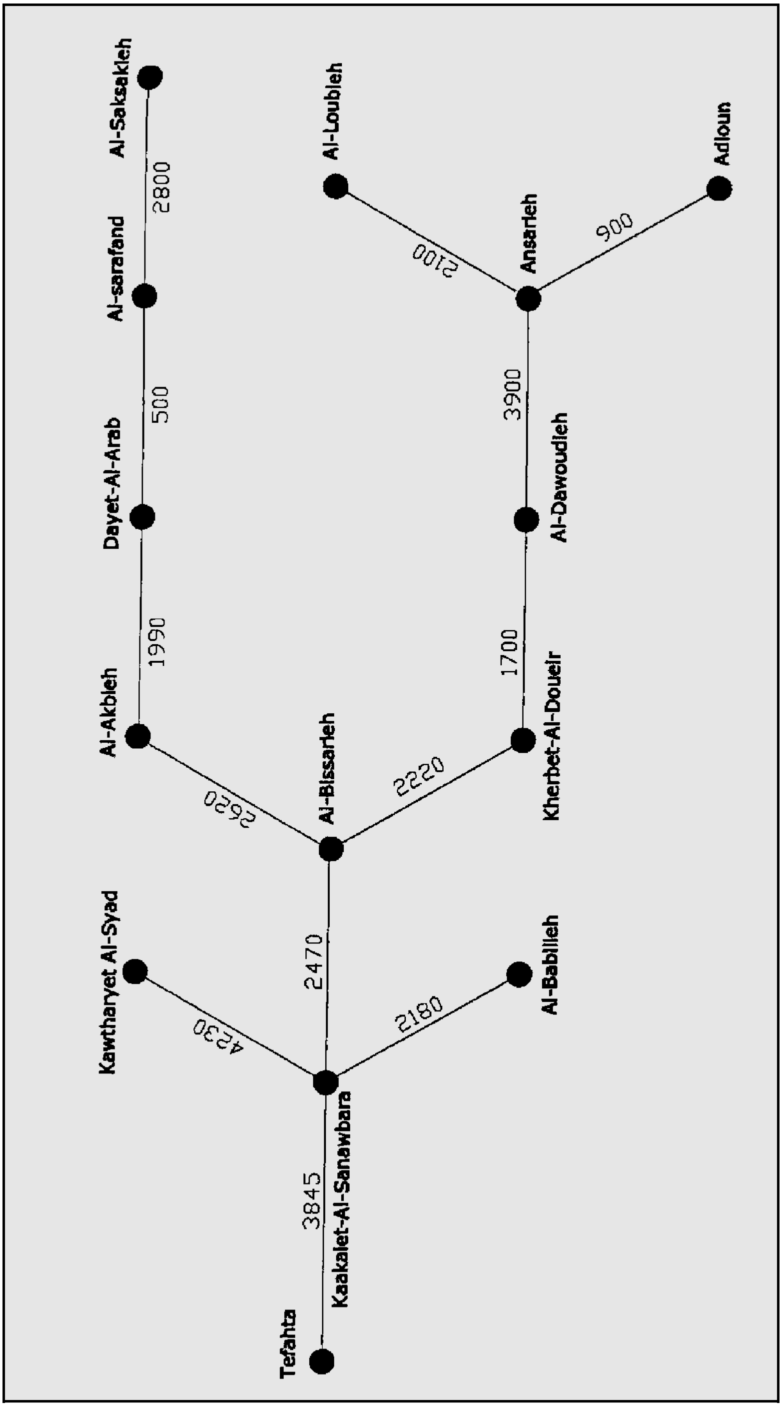
the total optimal network length is: 43935 + 31455 = 75390 meters,

and the total network length in the existing study is: 65495 + 40535 = 106030 meters.

The results show that the use of the MST network will lead to 30640 (= 106030 —75390) meters reduction of water pipes.

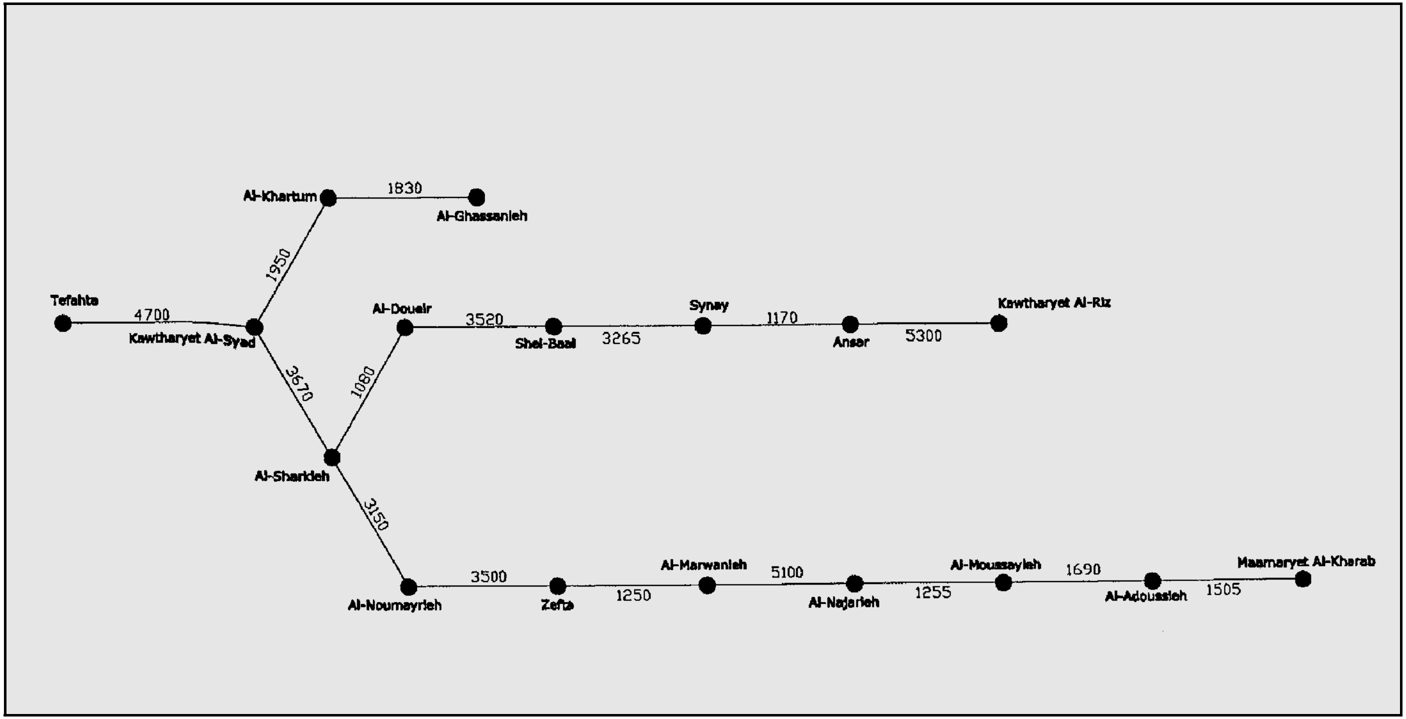
Figure 4

Graphical Representation of the MST Network of Water Allocation by Gravity



Note: All numbers are in meters.

Figure 5
Graphical Representation of the MST Network of Water Allocation by Pumbing



Note: All numbers are in meters.

Conclusion

The Municipal Waters engineers did a previous study that has two separate networks: one for towns provided with water by the force of gravity and another one by pumping. These two networks must operate together to nourish the 29 towns by water. Similarly, the MST solution, which is the optimal solution, has also two separate networks, one for gravity and the other one for pumping. The solution given by the Municipal Waters office is different from the MST solution. The major difference is that the total distance for each network, obtained in the MST solutions, is shorter than that of the corresponding one in the Municipal Waters study. Since the objective of this study is to reduce the total distance in each network, which means a reduction in the length of water pipes to be installed, The MST solution is the less expensive because it reduces the length of water pipes to be installed in the two networks by a total equal to 30640 meters.

Furthermore, if we look at the two optimal networks as a whole network we find that we can profit from the pumping network to reduce the cost of the gravity network. This is possible by substituting the branch Tefahta ˜ Kaakaiet-Al-Sanawbara by the branch Tefahta ˜ Kawtharyet-Al Syad. Although this procedure will increase the gravity network length, it will reduce the total whole network cost. The reason of this statement is that the branch Tefahta ˜ Kawtharyet-Al Syad is already in the pumping network, so the difference between the two branches (Tefahta ˜ Kaakaiet-Al-Sanawbara and Tefahta ˜ Kawtharyet-Al Syad) is 855 meters which will be an increase in the cost of equipment, i.e. an increase in the distance or the length of water pipes by 855 meters. As a result, the difference between the optimal networks and the actual study will be 29785 (=30640 - 855) meters.

This amendment will result in a reduction of 3845 meters of digging which means a reduction in the cost of digging.

In this study, we focused on the obtaining of a total minimal distance for the two networks (gravity and pumping). The amendment on the network was suggested by the Municipal Waters engineers who supposed the total costs which included the cost of digging and other costs would be less if they introduced this change in the network. Except this little change, the MST solution was adopted and implemented without any other modifications and the project of nourishing the 29 towns with water was executed and has already started to function.

Thus, the best solution for this problem is to adopt the minimal spanning tree networks that reduce to the minimum the cost of connecting all towns to the main station.

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الملخص

نموذج لشبكة توزيع حصص المياه باستخدام "Minimal Spanning Tree"

وداد عبدالحسن سعد روضة مصطفى شبو
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تهدف هذه الدراسة إلى تطبيق أحد الأساليب الكمية على عملية الحصول على شبكة تمديدات مثلى للمياه لـ ٢٩ قرية، تقع في جنوب لبنان، على أن يتم تغذيتها من عدة آبار، تقع جميعها في قرية واحدة، تدعى تفاحتا. ونظراً للطبيعة الجغرافية في تلك المنطقة، فقد تم تقسيم القرى إلى قسمين: أحدهما سيتم إيصال المياه إليه عن طريق الضخ، والآخر عن طريق الجاذبية. ومن هنا ظهرت الحاجة إلى وضع شبكتين للمياه. وبناءً عليه، تم استخدام التقنية المعروفة باسم Minimal Spanning Tree، تلك التي تمكننا من الحصول على أقصر شبكة لربط القرى العائدة لكل قسم، بالينابيع الموجودة في قرية تفاحتا. هذه الدراسة العلمية مكنت القيمين على المشروع من توفير مبالغ كثيرة بالمقارنة مع دراسة كان قد أعدها فريق من المهندسين العاملين في هذا المشروع.

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