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THE OPTIMIZATION OF PRODUCTION PLANNING AT AN INDUSTRIAL COMPANY

Key Words

Linear Programming; Planning; Optimal Solution; Product Mix; Algorithm; Industry.

Abstract

Two linear programming models for the problem of determining optimal planning strategies for an industrial production are presented in this paper. These models are established for two departments in a factory: foam and mattresses departments. The first model relates to finding a production planning for the foam department, whose products are either used in the mattresses department or sold directly to customers. The second model is concerned with the production of mattresses. An integer linear program is established to take into account the nature of the output of this department. Solutions are obtained, using the simplex algorithm, that does not only result in greater expected revenue compared to the previous period which had the same data, but also are less costly. The implementation of the optimal solutions leads to an increase in the revenue which equals up to 28.13% per period, without any additional costs. Sensitivity analysis for the optimal solution of foam production is developed to show possible changes in the optimal solution as a result of making changes in the original model.

Introduction

Since the advent of the industrial revolution, the world has seen a remarkable growth in the complexity of organizations. An integral part of this revolutionary change has been a tremendous increase in the division of labor and segmentation of management responsibilities in these organizations.

This increasing specialization has created new problems. A related problem is that as the complexity and specialization in an organization increase, it becomes more and more

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difficult to allocate its available resources to its various activities in a way that is most effective for the organization as a whole.

These kinds of problems and the need to find a better way to solve them provided the environment for the emergence of operations research (Hiller and Lieberman, 1990).

In this paper we develop two models: a linear programming model and an integer linear programming model. These techniques, which are among the efficient tools in operations research, are used to improve the raw material handling system and logistics of operations, for an industrial factory, called Fomaco, in order to maximize its total revenue.

This improvement will lead to two outcomes. Firstly, it will give a production planning that maximizes the total revenue of the company. Secondly, it will allow greater flexibility and enable the system to cope with the potential changing of the production strategy of the factory, without any additional costs.

The solution to this problem has been obtained by using the simplex method developed by George Dantzig in 1947.

This study is organized as follows: In the following section, the production system of concern is described, emphasizing the products, its components and the production process. After description of the real system, a model is constructed representing the production in the company. Analysis of the model is then explained followed by interpretation of the results obtained. The final section includes our concluding remarks and suggestion for future work.

Description of the Production System

Fomaco factory is one of the biggest producers of petrochemical products in Lebanon. The factory, which is located at Bekaa over an area of 9200m^2, was founded in 1986. Since then, it has continuously increased its production capacity and its share in the world of industrial market.

The factory contains two production departments:

1. Foam department.
2. Mattresses department.

The production process in the two departments is based mainly on the use of raw materials that are mostly from petrochemical origin. Furthermore, there is integration between the two departments in a way that the outputs of the foam department are the inputs of mattress department. The foam department operates independently from the other department and satisfies the demand of both the market and the mattresses department.

Table 1
The Sizes of the Mattresses in Both Groups of Mattresses

Width (cm)	90	100	110	120	160	165	170	175	180
Length (cm)	195	195	195	195	195	195	195	195	195

The two basic product types need a variety of mechanical performance and have different final output according to which their operation routings differ.

Product Mix

The factory produces two kinds of products:

1. Foam: there are ten different kinds of foam: X, C18, C20, and BB. SF, B, BS, EX, EXS, and SL.
2. Mattresses: there are two main groups of mattresses: foam mattresses and spring mattresses.
   * Foam mattresses: there are five types of foam mattresses (Dolphin, High, Lux, Superlux and prestige), and in each type there are nine sizes.
   * Spring mattresses: there are seven types of mattresses (Nice, Fine, Royal, paradise, Regency, Sovereign, and Eminence), and in each type there are nine sizes.

The sizes of the mattresses in the two groups of mattresses are summarized in table (1).

Production Flow and Product Components

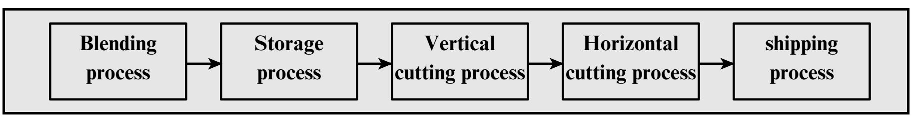
We will describe foam and mattresses productions and give the details of the components of each product. The production process of all kinds of production is mostly done automatically on machines.

1. Foam production

Fomaco manufactures ten different kinds of foam and each block from any kind must go through five processes. The flow production of foam production is illustrated in figure (1).

The blending process consists of mixing several petrochemical components with water to produce one block of foam. The blending process is a critical process and any mistake results in the destruction of the whole mixture. In this step, essential ingredients are blended so that the block of

Figure 1
Flow Production of Foam



foam obtained contains a specific amount of each ingredient. This procedure is done in a computerized machine, called casting machine, which programs the amounts of raw materials needed for each block and type of foam. This machine is related to the warehouse by a 50-meter line that transports the foam to it automatically. At this stage, the storage process starts. Then, the obtained block of foam is trimmed and cut horizontally with an electronic saw. It can carry out three blocks of foam at once. The vertical cutting is run with another electronic saw, at desired measures. This saw can execute one block of foam at the time. The process of shipment is very delicate and needs skilled employees because of the fragility of the foam.

2. Mattresses production

Fomaco produces two groups of mattresses and each group contains different types and sizes. Mattresses from each type are processed successively through six operations that are indicated in figure (2).

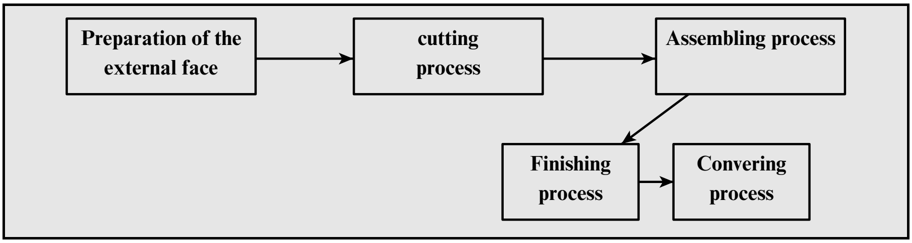
The first process consists of the preparation of the external face of a mattress, which is composed of 4 materials, placed one above the other as follows: textile, Dacron, foam, and lining. Then they are sewed on a particular machine. In the second process, the obtained external face is cut into appropriate sizes for different kinds of mattresses. For each mattress, we need two faces and four sides. In the third process, all the necessary things for getting a mattress are put together. For the first type of mattresses, foam is put between the two faces. For the second type of mattresses, a chassis is prepared and covered with foam and then the faces and sides are placed appropriately. The fourth process consists of sewing all the parts of the mattress. As a final process, the obtained mattress is covered by nylon and stored.

Aspects of the Problem

Our system is a mixture of semi-finished and finished manufacturing. This has led us to the development of

Figure 2

Flow Production of Mattresses



two models. In the first model, the foam production process is modeled separately from the rest of the system as a linear programming model. The reason for that is that the foam department operates independently from the second department (mattresses department) and the factory sells a big quantity of the foam as semi-finished product. This model is mainly used to estimate the optimal quantity to produce of each type of foam in order to satisfy the demand of the factory and customers and to maximize the revenue of the foam department. The second model is concerned with the finished products of mattresses. It includes the production process of mattresses. Since the items in the mattresses department cannot be fractioned we were obliged to establish an integer linear programming model that is specific for this kind of problems. In this model we use the semi-finished product of the first model as raw material in the second. The purpose of modeling in the second case is two folds: (1) determine the optimal quantities to be produced of each type of mattresses with respect to the capacity of production, the availability of raw materials, the available machine hours, and the yearly market demand; (2) maximize the revenue of this production line.

Modeling the production of Fomaco factory requires extensive data analysis to be able to determine input quantities. One year's data have been taken into account to determine the least quantity demanded of each type of products. We used the data of the year 2001. These data are used from year to year in the factory without significant changes.

Fomaco mixes amounts of different available raw materials to produce one block of foam used in the mattresses department or sold to customers. Each kind of foam differs not only in price but in composition as well. Table (2) indicates the amounts of ingredients found in each kind of foam, taking into account that these quantities are used in one minute of operating machines. The factory cannot produce more than three blocks a week in all because it is the maximum required in the share market of Fomaco.

The accompanying table (2) contains all relevant information concerning production times required as economic times to produce a block of foam of each kind, raw materials needed to produce an amount of foam per minute, along with revenue per block produced and minimum yearly production requirement to fulfill the minimum quantities demanded.

The amount of a block is obtained by multiplying its economic time by the one-minute output on machines of this kind of foam.

Table 2

The Amounts in kg of Raw Materials Needed in the Production of Each kind of Foam During one Minute of Time on the Machines

Raw materials a_{ij} / Products x_j		Polyol (kg) (a_{1j})	TDI (kg) (a_{2j})	Water (kg) (a_{3j})	Mathylene Chloride (kg) (a_{4j})	Tegostab BF2370 (kg) (a_{5j})	Niax A1 (kg) (a_{6j})	Dabco 33LV (kg) (a_{7j})	Stannous Octoate (kg) (a_{8j})	Economic Time (mn) (t_j)	Demand (block)** (d_j)	(u_j)	Amount of foam/ minute*	Price/ Block ($) c_j
X	(d_1)	112.11	68.32	5.72	8.63	1.29	0	0.09	0.37	85	8	-	584.28	42219.33
C18	(d_2)	140.14	81.01	6.73	0	1.43	0	0.11	0.3	90	17	25	546	54054.00
C20	(d_3)	140.14	73.69	6.03	0	1.39	0	0.11	0.28	70	4	10	479.16	40249.44
BB	(d_4)	152.15	65.71	5.17	0	1.44	0.1	0.12	0.26	107	21	-	395.58	57141.45
B28	(d_5)	152.15	65.12	4.49	0	1.4	0.21	0.12	0.21	80	7	-	348.15	45120.24
BS28	(d_6)	152.16	63.58	4.56	0	1.4	0.2	0.12	0.22	62	2	-	348.95	33534.10
SF	(d_7)	152.15	51.58	3.88	7.91	1.46	0.38	0.12	0.28	66	6	-	375.36	38399.32
EX	(d_8)	152.15	51.44	3.43	0	1.32	0.58	0.12	0.19	102	11	-	256.1	50938.29
EXS	(d_9)	152.16	48.97	3.43	0	1.32	0.59	0.12	0.19	75	7	-	251.94	35901.45
SL	(d_{10})	152.15	37.67	2.36	3.35	1.26	1.89	0.12	0.17	80	4	-	205	42968.00
Availability (kg)		12155110 (e_1)	714420 (e_2)	abundant	17625 (e_3)	26600 (e_4)	2100 (e_5)	1500 (e_6)	4800 (e_7)					

Source: Fomaco Factory

*The unit of foam is 1cm of height, 1m of width and 2m of length.

** The amount of a block is obtaired by multiplying the out put of one - minute production time of certain product by its economic time.

d_j: lower - bound

U_j: Upper - bound

As noticed before, each type of mattresses is produced in nine sizes. The ingredients and the prices are proportionally distributed according to the sizes in each type of mattresses. Because of that, if we choose one size in each type and we do the study, we will get the optimal quantities to produce of each type. In order to simplify the problem and make it more convenient, we took the standard size 120 for all types of mattresses and used it for this study. In fact, this size is in demand more than the other sizes. The results of the study will allow the manager to have an idea of the type he should increase its production whatever the sizes are.

The following table (3) indicates the amounts of ingredients found in each type of mattresses. It also contains the least yearly demand for mattresses and the price of each type of mattress.

Times needed in these processes are not readily available in the factory. We collected data in the real system by observing a cycle of production and calculating the time in minutes for each operation. These times are illustrated in table (4).

Research Objectives

The factory seeks to determine the amounts of products to be produced in each department (foam and mattresses departments) to maximize (increase as much as is feasible) the total gross income (in dollars) while satisfying the constraints of demand, available machine hours, and raw materials usage.

Model construction and development represent the crucial step in the implementation of an optimal solution to this problem. Thus, an accurate and exact solution to the model will not be useful unless the model itself provides an adequate representation of the true decision situation.

The construction of a mathematical model can be initiated by answering the following three questions?

1. What are the variables of the problem?
2. What constraints must be imposed on the variables to satisfy the limitations of the modeled system?
3. What is the objective (goal) that needs to be achieved to determine the optimum (best) solution among all the feasible values of the variables?

An effective way to answer these questions is to identify the problem and define all the sets, variables and parameters needed for its solution.

Theoretical Background

There are two aspects of the problem to be addressed. One is the

Table 3

Amount of Raw Raterials Used in the Production of Each Kind of mattresses, Times Needed in Each Operation and Availability of Raw Materials

Products Y_k Raw materials e^*_{rk}		Dolphin (y_1)	High (y_2)	Lux (y_3)	Super Lux (y_4)	Prestige (y_5)	Nice (y_6)	Fine (y_7)	Royal (y_8)	Paradise (y_9)	Regency (y_{10})	Sovereign (y_{11})	Eminence (y_{12})	Availability (f_r)
Wire	1.3 mm (e_{1k})*						1.35	1.35	1.35	1.35	1.35			4947
(kg)	4.5 mm (e_{2k})						1.56	1.56	1.56	1.56	1.56			5110
	2.2 mm (e_{3k})						6.83	6.83	8.40	8.40	8.40			20881
	10 mm (e_{4k})						0.20	0.20	0.20	0.20	0.20			1848
(Kg)	Felt 1 (e_{5k})								5.40	5.40				3923
(Kg)	Felt 2 (e_{6k})							6						2536
Unit	Snaps (e_{7k})						200	200	250	250	250			414450
Foam	C18 (g_{8k})	17.52	1.92	1.92	1.92	1.92	6.72	6.72	8.48	2.48	2.48	1.92	1.92	40197.8
(unit)**	BB (e_{8k})		15.6							7.2	7.2	2.5	2.5	37398
	SF ($e_{10,k}$)				7.2	12								2768.24
	EX36 ($e_{11,k}$)			16.8										20226.16
	SL ($e_{12,k}$)				12	12						12	12	4836
Unit	Spring ($e_{13,k}$)								6	6	6			8170
(Kg)	Dacron($e_{14,k}$)		0.70	0.70	0.70	0.70		0.70	0.70	0.70	0.70	0.70	0.70	3026
(Kg)	Lining ($e_{15,k}$)		0.80	0.80	0.80	0.10		0.80	0.80	0.80	0.10	0.10	0.10	13000
Textile	A (m) ($e_{16,k}$)	3.25					3.25							5000
(m)	B (m) ($e_{17,k}$)		3.25					3.25						18339
	C (m) ($e_{18,k}$)			3.25					3.25					5575.5
	D (m) ($e_{19,k}$)				3.25	4.5				3.25	4.5			1315.5
	E (m) ($e_{20,k}$)											3.4	5	517.6
(Kg)	Glue ($e_{21,k}$)	0.20	0.20	0.20	0.20	1		0.15	0.20	0.20	1	1	1.50	2490
(Kg)	Nylon 1 ($e_{22,k}$)	0.40				0.84	0.42		0.42	0.42	0.84	0.84	0.84	1415
(Kg)	Nylon 2 ($e_{23,k}$)		0.33	0.33	0.30			0.33						4544
(m)	Treads ($e_{24,k}$)	110	110	110	110	220	110	110	110	200	220	200	220	828530
Unit	Label ($e_{25,k}$)	2	2	2	2	2	2	2	2	2	2	2	1	25586
(m)	Twine ($e_{26,k}$)	13	13	13	13	55	13	13	13	13	55	26	55	83844
Unit	Plastic corner ($e_{30,k}$)			4	4	4			4	4	4	4	4	16140
(Kg)	Burlap ($e_{31,k}$)						0.40							1260
(Kg)	Cotton ($e_{32,k}$)						6							5866
Unit	Chassis ($e_{32,k}$)											1	1	234
Price/unit ($) ($p_k$)		42	59	84	105	135	41	55	68	84	118	210	252	
Minimum yearly quantity demanded (m_k)*		700 (units)	1701 (units)	892 (units)	160 (units)	34 (units)	595 (unit)	292 (units)	532 (units)	88 (units)	12 (units)	14 (units)	14 (units)	

Source: Fomaco Factory.

* k = 1,2,,12 and r = 1,2,....,32..

**The unit used in foam is 1cm of height, 1m of width and 2m of length.

Table 4
Time in Minutes Needed for Each Process in the Production of Each type of Mattresses

		Processes of production h_{kn}*				
Raw materials	Quantity to be Produced y_k	Process 1 (h_{k1})	Process 2 (h_{k2})	Process 3 (h_{k3})	Process 4 (h_{k4})	Process 5 (h_{k5})
Dolphin	y_1	2.00	1	5	5	3
High	y_2	2.30	1	5	5	3
Lux	y_3	2.30	1	5	5	3
Super Lux	y_4	3.30	1	5	5	3
Prestige	y_5	3.30	1	10	10	3
Nice	y_6	2.00	1	6	5	3
Fine	y_7	2.30	1	6	5	3
Royal	y_8	3.30	1	6	5	3
Paradise	y_9	3.30	1	6	5	3
Regency	y_{10}	4.00	1	10	10	3
Sovereign	y_{11}	4.00	1	20	15	3
Eminence	y_{12}	4.00	1	30	30	3
Availability	s_n*	144000 (mn) (s_1)	144000 (mn) (s_2)	144000 (mn) (s_3)	144000 (mn) (s_4)	144000 (mn) (s_5)

Source: Fomaco Factory.
*n = 1,2,3,4,5 and k = 1,2, ...,12.

production planning of foam department that maximizes its revenue. The other is the production program of the mattresses department, which also maximizes its revenue.

To better understand the structure of the problem and gain some initial insight, we first consider the problem of planning production of foam and then we will treat the production program of the mattresses department. The problem will be formulated and resolved using linear and integer linear programming. They are powerful techniques for dealing with the problem of allocating limited resources among competing activities. The approach of linear programming is that of scientific method. In particular, the process begins by carefully observing and formulating the problem and then constructing a scientific (typically mathematical) model that attempts to abstract the essence of the real problem. This model is sufficiently precise representation of the essential features of the situation [Hillier and Lieberman, 1990; Render and Stair, 1994; Taha, 1997]. This technique has been used successfully in the solution of problems concerned with assignment of personnel (Moondra, 1976), (Anbiletal, 1991), industry (Kabroal, 1981) transportation (Harrison, 1979), aircraft (Anbil *et al.*, 1991), (Subramanien and Anbil *et al.*, 1994), financial institutions (Ben Dov, Hayer and Pica, 1992), (Saad and Chebbo, 2001), (Balbiar and Shaw, 1981), military (Peatre, 1987), assignment, (Breslaw, 1976), medical field (Hilal and Erikson, 1981), (Left, Dada, and Graves, 1986) production (Cabraal, 1981), agriculture (Glen, 1986), sports, (Zappe and Horowitz, 1993), and many other fields.

The Simplex method, developed by George Dantzig in 1947, is an efficient and reliable algorithm for solving linear programming problems. It also provides the basis for performing the various parts of post-optimality analysis very efficiently.

The Models

Linear and integer linear programming models of production planning in the two departments (foam and mattresses) are developed. These models determine the quantities to be produced in each department in order to maximize the total revenue. Taking into account the specifications of each department, the input data, used in these models, consist of the following.

Sets

* Foam

Raw material, $i = 1,2,...,8$.

Foam, $j = 1,2,...,10$.

* Mattresses

Mattress, $k = 1,2,...,12$.

Raw material, $r = 1,2,...,32$.

Process, $n = 1,2,3,4,5$.

Parameters

* Foam

a_{ij} amount of raw material i used in the production of foam of type j during one minute of operating machines.

b_i availability of raw material i during a year.

c_j price of a block of the foam of type j.

d_j least yearly demand of the foam of type j.

u_j upper bound (or limit) of the yearly demand of the foam.

* Mattresses

e_{rk} amount of raw material r used in the production of one mattress of type k.

m_k least yearly demand of the mattresses of type k.

p_k price per unit of each type k of mattresses.

h_{kn} time needed in the process n to produce one mattress of type k.

s_n availability of time on the machines during a year.

f_r availability of raw material r.

Variables

* Foam

x_j the number of foam blocks of type j to be produced during one year.

* Mattresses

y_k the number of mattresses of type k to be produced during one year.

Mathematical formulation

The mathematical models were formulated using the linear programming technique and are solved using simplex algorithm developed by George Dantzig. Two models are presented. The first model is for the production planning in the foam department and the second one is for a production program in the mattresses department.

1. The model of the foam production (MF) is as follows:

Objective function (to be maximized)

If we let z represent the total gross revenue (in dollars) of the foam department, the objective function is written mathematically as:

$$Max z = \sum_{j=1}^{j=10} c_j x_j \qquad (\alpha)$$

Constraints

Raw material constraint

$$\sum_{j=1}^{j=10} a_{ij} x_j \leq b_i \quad for\ i = 1, 2, ..., 8. \quad (1)$$

Demand constraint

$$\begin{aligned} x_j &\geq d_j \qquad for\ j = 1, 2, ..., 10 \\ x_j &\leq u_j \qquad for\ j = 2, 3 \end{aligned} \qquad (2)$$

Non-negativity constraint

$$x_j \geq 0 \qquad for\ j = 1, 2, ..., 10. \quad (3)$$

Constraint set (1) states that the amounts of raw materials used in the production of foam are equal or less than their availability. Lower and upper bounds on the production of all types of foam are represented by constraint set (2). These limits are determined by the least and highest yearly demand of each type of foam. These amounts are either sold to customers or used in the mattresses department. Constraint set (3) ensures that all the produced quantities are non-negative since we cannot produce negative quantities. The objective function (?) is a maximization of the total revenue z of the foam department.

2. The model of the mattresses production (MM) is as follows:

Objective function (to be maximized)

If we let w represent the total gross revenue (in dollars) of the mattresses department, the objective function is written mathematically as:

$$Max\ w = \sum_{k=1}^{k=12} p_k y_k \qquad (\beta)$$

Constraints

Raw material constraint

$$\sum_{k=1}^{k=12} e_{rk} y_k \leq fr \quad for\ r = 1,2,..,32 \quad (4)$$

Demand constraint

$$y_k \geq m_k \qquad for\ k = 1,2,..,12 \quad (5)$$

Time availability constraint

$$\sum_{k=1}^{k=12} y_k h_{kn} \leq sn\ for\ n = 1,2,3,4,5. \quad (6)$$

Non-negativity constraint

$$y_k \geq 0 \qquad for\ k = 1,2,...,12. \quad (7)$$

Constraint set (4) stipulates that the amounts of raw materials used in the production of mattresses department will not exceed their availability. In constraint set (5) a lower limit is established by the least market demand of mattresses of each type. Constraint set (6) place limits on the use of machines. The last constraint set (7) ensures the non-negativity of the quantities of mattresses produced. The amount produced of each product cannot be negative or less than zero. To avoid obtaining such a solution, the non-negativity restrictions are imposed. The objective function (β) establishes a maximization constraint w, which is constraint to be the sum of all possible revenues in the mattresses department.

Results /Analysis

Developing solutions of the above models is an important step in the quantitative analysis approach. Because we are using mathematical models, these steps require mathematical calculations. Powerful software, called Lingo 5.0, is used to solve this problem. It can handle problems with large functional constraints and variables. It is based on the simplex method and provides accurate solution to linear and some specialized nonlinear optimization problems.

Table (5) presents the different results pertaining to the two models. It shows the optimal amounts to be produced of each product in each department in order to maximize the total revenue of the factory. The first model, which is related to the foam department generates a revenue equals to $5414419.65 per year and the second one results in a revenue of $393 087.00 per year. As a result, the total revenue of the factory is the sum of these two revenues, which is equal to $5 807506.65 per year.

Table 5
Optimal Quantities of Foam and Mattresses to be Produced and their Revenues

	Mattresses department					Foam department			
Mattresses	Quantity to be produced		Price/mattress ($)	Revenue ($)	Foam	Quantity to be produced		Price/block ($)	Revenue ($)
	Variable	(unit)				Variable	Block		
Dolphin	Y_1	943	42	39606	X	X_1	18.29547	42219.33	772422.486
High	Y_2	1790	59	105610	C18	X_2	25.00000	54054.00	1351350.000
Lux	Y_3	1202	84	100968	C20	X_3	10.00000	40249.44	402494.400
Super Lux	Y_4	160	105	16800	BB	X_4	22.00000	57141.45	1291343.628
Prestige	Y_5	98	135	13230	B28	X_5	7.00000	45120.24	315841.680
Nice	Y_6	595	41	24395	BS28	X_6	2.00000	33534.10	67068.200
Fine	Y_7	292	55	16060	SF	X_7	6.00000	38399.32	230395.920
Royal	Y_8	532	68	36176	EX	X_8	11.00000	50938.29	560321.190
Paradise	Y_9	88	84	7392	EXS	X_9	7.00000	35901.45	251310.150
Regency	Y_{10}	15	118	1770	SL	X_{10}	4.00000	42968.00	171872.000
Sovereign	Y_{11}	130	210	27300	Total				5414419.654
Eminence	Y_{12}	15	252	3780					
Total				393087					

Table 6
Optimal and Current Production Plans

Department	Revenue of the production plan of 2001 ($)	Revenue or the optimal production plan ($)
Foam	4 214 449.99	5414419.65
Mattresses	317 984.00	393 087.00
Total	4 532 433.99	5 807 506.65

As noticed before, the data used in this study refer to the year 2001. The factory used the same raw materials and machinery that are applied in this study. Table (6) presents the optimal revenue pertaining to optimal solutions and the revenue resulting from the current production strategy in the factory.

The total revenue of the factory during 2001, resulting from the application of its own production planning was $4 532 433.99.

A simple comparison between the revenue of the optimal solution and the revenue of the current situation shows the importance of the implementation of scientific technique in management. The optimal solution gives an increase in the revenue equals $1275072.66, which represents 28.13% increase in the revenue.

The rate of Increase in the revenue

$$= \frac{5807506.654-4532433.990}{4532433.990} \times 100 = 28.13\%$$

Our models give solutions that are much better than the current solutions, resulting in potential revenue in excess of $1 275 072.664 over the year 2001. Since the cost of using the models presented in this paper is virtually zero, the increase in revenue corresponds to real profits.

We carried out sensitivity analysis for the optimal solution of foam production. This information shows how the optimal solution and the value of its objective function change, given changes in various inputs to the problem. We used LINGO to compute ranges for the objective function coefficients, right-hand-side values and shadow prices.

* Changes for the objective function coefficients

In general, if the objective function coefficient of a single variable is changed within a certain range, then the values of the decision variables will not change. However, the value objective function may change. The information in table (7) tells us the effect of changing a single price of a product.

The products X, C18, and C20 can individually have any prices greater than zero without altering the optimal values of the decision variables. This shows very significant flexibility of these coefficients. Similarly, as long as the price of a block of foam BB is greater or equal,

Table 7
Optimal Solution Without Upper and Lower Limits

Product	Variable	Current coefficient (c_j)	Ranges
X	x_1	42219.33	$0 < c_1$ (for x_1) $\leq \infty$
C18	x_2	54054.00	$0 < c_2$ (for x_2) $\leq \infty$
C20	x_3	40249.44	$0 < c_3$ (for x_3) $\leq \infty$
BB	x_4	57141.45	$28936.68 < c_4$ (for x_4) $\leq \infty$
B28	x_5	45120.24	$0 < c_5$ (for x_5) ≤ 89717.42
BS28	x_6	33534.10	$0 < c_6$ (for x_6) ≤ 66220.00
SF	x_7	38399.32	$0 < c_7$ (for x_7) ≤ 163982.30
EX	x_8	50938.29	$0 < c_8$ (for x_8) ≤ 315933.50
EXS	x_9	35901.45	$0 < c_9$ (for x_9) ≤ 236309.30
SL	x_{10}	42968.00	$0 < c_{10}$ (for x_{10}) ≤ 82281.40

then 28936.68, the current production mix solution will be optimal. It follows that; the remaining coefficients can be changed unilaterally, over their ranges, without affecting the character of the optimal solution. However, these coefficients are less flexible than the first three ones.

* Changes for the right-hand-side values

A shadow price is the change in value of the objective function from an increase of one unit of a scarce resource. The addition of one unit of an abundant resource does not change the objective function value. All these changes dealt with the right-hand-side of the constraints. The right-hand-side of a single constraint can be changed within a specified range to keep the same shadow price. However, the values of the decision variables and the total revenue of the solution may change. All the necessary information about shadow prices and ranges are given in table (8).

Table 8
Shadow Prices and Ranges of the Raw Materials

Raw materials	Current availability (bi)	Slack or surplus	Shadow price	Type of the resource	Range
Polyol	12 155 110.00	10 735 830.00	0.00	Abundant	$149\,270 \leq b_1 \leq \infty$
TDI	714 420.00	55 169.58	0.00	Abundant	$659\,250.42 \leq b_2 \leq \infty$
Mathylene Chloride	17 625.00	0.00	57.55	Scarce	$10\,072.76 \leq b_3 \leq 24\,593.87$
Tegostab BF2370	26 600.00	12 808.34	0.00	Abundant	$39\,408.34 \leq b_4 \leq \infty$
Niax A1	2 100.00	0.00	5 340.32	Scarce	$2\,082.89 \leq b_5 \leq 2\,183.96$
Dabco 33LV	1 500.00	379.72	0.00	Abundant	$1\,120.27 \leq b_6 \leq \infty$
Stannous Octoate	4 800.00	2101.812	0.00	Abundant	$2\,698.19 \leq b_7 \leq \infty$

There are two scarce resources, Mathylene Chloride and Niax A1. The range over which the addition of one unit of Mathylene Chloride will cause the objective function to improve by $57.55 to a value of

$5414477.20, is between 10072.76 and 24 593.87. Similarly, As long as the amount of Niax A1 is greater or equal to 2082.89 and less than or equal to 2 183.96, the addition of one unit of this resource will increase the optimal solution by $55 169.58 to a value of $5469589.23 dollars. The remaining resources can increase indefinitely without altering the problem's solution.

Linear programming is a mathematical technique used to help management decide how to make the most effective use of an organization's resources. This approach starts with data that are manipulated and processed into meaningful information. This processing and manipulation is the heart of quantitative analysis.

In addition to quantitative analysis qualitative factors should also be considered. The weather, legislation, new technology, and so on may all be factors that are difficult to quantify. Because of the importance of qualitative factors, the role of quantitative analysis in the decision-making process can vary. When there is a lack of qualitative factors and when the problem, model, and input data remain the same, the results of quantitative analysis can automate the decision-making process. In most cases, however, quantitative analysis will be an aid to the decision-making process. The results of quantitative analysis will be combined with other (qualitative information) in making-decisions. Moreover, an optimal solution resulting from a quantitative analysis should be accompanied with a marketing study and advertisement since such a solution, suggest most of the time, increasing the production of certain products and decreasing the others.

Conclusion

In this paper we have developed two linear programming models for managing the production in two different departments pertaining to the same factory.

We develop a first model to determine the best quantities of foam to produce in order to maximize the revenue of this department. The second model was a special type of linear programming models, called an integer linear programming model. The construction of this model is identical with the first one, but the only difference is that the decision variables have to take an integer value in the final solution. This model seeks to maximize the revenue of the mattresses department.

We saw that our models solved the existing problem much better than the current policy, resulting in potential revenue increase of $1275072.66 per year. This represents 28.13% increase in the total revenue. The cost of the optimal solution implementation is zero. As a result, the increase in revenue corresponds to real profits for the factory. This result demonstrated the importance of using linear programming model in decision-making. It leads, under the same circumstances, to an increase in the revenue estiwated at 28.13% per period.

We supplemented the optimal solution of foam production with sensitivity analysis, which recognized the sensitivity of the optimal solution to changes in the data used to build the model.

The need to efficiently allocate limited and expensive resources, in each company, necessitates the employment of new strategies based on scientific methods and techniques such as linear programming.

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الملخص

التخطيط الإنتاجي الأمثل لإحدى الشركات الصناعية

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تتناول هذه الدراسة تطبيق إحدى وسائل الأساليب الكمية على إنتاج إحدى الشركات الصناعية في لبنان. ولقد قمنا باستخدام أسلوب البرمجة الخطية على قسمي الإنتاج في هذه الشركة: قسم الإسفنج وقسم الفرشات، وذلك لإيجاد خطة إنتاج مثلى تعمل على زيادة دخل الشركة، ضمن المعطيات الحالية الموجودة لديها. وتم التوصل، من خلاله، إلى نتائج إيجابية لجهة زيادة العائد دون أن يرافق ذلك أية زيادة في التكاليف؛ حيث إن العائد من الخطة الإنتاجية المثلى، التي حصلنا عليها عن طريق استخدام اللوغاريتمات السمبلكسية، يزيد عن العائد الناتج عن تطبيق الخطة الإنتاجية المطبقة خلال عام ٢٠٠١ بحوالي ١٢٧٥٠٧٢,٦٦ دولار أميركي، أي ما نسبته ٢٨,١٣٪ وذلك ضمن نفس المعطيات المستخدمة في تلك السنة. كما أنه قد أجري تحليل حساسية للخطة المثلى لإنتاج الإسفنج، وذلك لإظهار مدى تأثرها بتقلبات الأسعار، وتغير كميات المواد الأولية الداخلة في الإنتاج.

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