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THE COST OF PERFECT INFORMATION FOR ESTIMATING THE EXPONENTIAL LOCATION

Key Words

Fisher information; Loss functions; Opportunity cost; Squared error loss; Triple sampling

Abstract

In this study the concept of perfect information is implemented to combine both point and confidence interval estimation problems in one decision framework to achieve maximal use of the available sampling information. Specifically, point estimation of the exponential location is viewed in the larger context of interval estimation, since the information required to determine optimal point estimators, per se, is inherently unknowable. Triple sampling procedure is shown to be a reasonable technique for obtaining the sample information required to satisfy specific objectives within this unified decision framework. We also extend our investigation to discuss the ability of the triple sampling confidence interval to detect shifts in the location parameter of the exponential distribution.

Introduction

Classical inference accommodates many impaired procedures. A classical point estimator, for example, focuses exclusively on one possible value of the unknown parameter under investigation. This value is unlikely to prove adequate or satisfactory if reviewed in isolation. Many authors believe that by attaching the standard error to the point estimate then the inference is accomplished see for example, Efron and Hinkley's (1978: 458). Knowing that the standard error provides no indication about the reliability of inference leaves us wondering about the above suggestion. The bare fact is, regardless of the method we use to perform inference, our conclusions are uncertain since our knowledge is subjected to random variation. The only useful inferential procedures are those for which the degree of uncertainty can be measured in some probability sense. It would be indeed, an accomplishment if we could assess the uncertainty involved in our knowledge or belief about the targeted parameter. We must also demand to know what kind of precision we have achieved. We quote from Cox and Hinkley (1992: 251) regarding point estimation methodology"... no completely satisfactory solution can, however, be given until fairly explicit information is available both about the quantitative consequences of the errors of estimation and also

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about the relevant information other than data under analysis". Therefore, this overemphasis of point estimation can be viewed within a set of plausible values, determined from the data, within which the unknown parameter is thought to lie with a predetermined confidence along the lines of Cox and Hinkley above.

In the area of multistage sampling, the literature has considered two main methodologies to tackle estimation problems either point estimation, where a specific cost function is assumed to assess the anticipated risk; or fixed width confidence interval estimation to attain a given nominal coverga value. These two methodologies have been treated separately in a given situation. In the present study, we combine these two methodologies in a single unified decision framework to utilize the available sample information to treat both problems simultaneously. This has been accomplished while implementing the concept of the cost of perfect information introduced by Hamdy et al. (1996). Specifically, if a point estimator for the exponential location parameter is required, and that the risk associated with a given loss function is to be minimized to determine the required optimal sample size. Would it be appropriate to use the same sample information, determined by the previous criterion, to construct a satisfactory fixed width confidence interval with a specified coverage probability for the exponential location? Answering such a question requires a deep understanding of the structure of the assumed loss function on the bases of the perfect information.

In the following sections we discuss the role of the cost of perfect information in performing infrerence while estimating the location parameter of the exponential distribution.

Estimating the location parameter of the exponential distribution

Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables having the negative exponential distribution of the form

$$f(x;\mu,\sigma) = \sigma^{-1}e^{-(x-\mu)/\sigma} \quad I(x > \mu), \sigma > 0, \qquad (1)$$

Where I $(\cdot)$ is the usual indicator set function. Both μ and σ are assumed unknown but finite. The main objective in this study is estimation of μ. Suppose further that a random sample $X_1, X_2, \ldots, X_n$ of size n (≥ 2), from the distribution in (1), is available. Then, we propose to use the sample measures

$$X_{n(1)} = \min_{1 \leq i \leq n} X_i$$

and $\hat{\sigma} = (n-1)^{-1}\sum_{i=1}^{n}(X_i - X_{n(1)})$ as the point estimators for μ and σ respectively. Moreover, we assume that the loss incurred in estimating μ by $X_{n(1)}$ can be reasonably represented by the following squared error loss function with a linear sampling cost

$$L_n = A(X_{n(1)} - \mu)^2 + cn \qquad (2)$$

The leftmost component on the right hand side of (2) reflects the cost of estimation, while the rightmost component reflects the cost of sampling, c is the known cost of an observation, and $A > 0$ is assumed to be a "known weighting" constant. See for example, Robbins (1959), Starr (1966), Starr and Woodroofe (1969, 1972), Woodroofe (1977, 1985), Ghosh and Mukhopadhyay (1979, 1980), Vardi (1979), Chow and Martinsek (1980), Chow and Yu (1981), Mukhopadhyay (1985), Mukhopadhyay et al. (1987), Martinsek (1988), Hamdy (1988), Hamdy et al. (1988), Hamdy and Costanza (1995), and Hamdy et al. (1995). However, optimal point estimators determined solely through minimizing the risk associated with (2) are essentially useless for statistical inference. The essence of this philosophical dilemma is that the usual characterization of the constant A stated above is impractical. To show that, we write the expected loss (risk) of estimation as

$$R_n = E(L_n) = A\sigma^2/n + cn. \tag{3}$$

Hence, the optimal sample size required to minimize the risk in (3) is given as

$$n^* = \sqrt{A/c}\,\sigma. \tag{4}$$

From which we write

$$n^*/\sigma = \sqrt{A/c}. \tag{5}$$

Most of the work done in the area of multistage point estimation of a parameter has involved the assumption of complete knowledge of the value of A. As we have shown in (5), for the case of estimating the exponential location parameter with squared error loss in (2), such claim undoubtedly implies that the decision maker knows the value of the ratio of the optimal sample size to the population standard deviation (n^*/σ). Thus, the practice of prespecifying the value of A in point estimation applications must eventually lead to "fictitious optimal" sample sizes. This is because their determination through (4) is based on a self-fuifilling criterion which is unrelated to and thereby artificially and mysteriously restricts the population of inference. In effect, the sampled population is not the target population that needs to be explored freely through the sampling procedure in the first place. Further, it is also common in the literature to leave the interpretation of the constant A in (2) to the subjective judgment of the decision maker. While utility theory descriptions of decision makers who are either "risk averters" or "risk neutral" or "risk seekers" neither make this practice more objective nor do they go much farther in shedding light on the interpretation of A in the loss function. Is this a conceivable assumption in the context of obtaining a point estimator for μ via multistage sampling, the basic premise of which is that σ is completely unknown?

Most authors have specified or characterized A in (2) in a way that has essentially ignored objective practical and statistical meaning in modeling the estimation cost. For example, Starr and Woodroofe (1972), described A as being "chosen in advance of experimentation to express the weight that the experimenter assigns to estimation error relative to sampling costs". Although the characterization might appear to be an objective definition, the phrase "chosen in advance" still involves elements of subjective assignment and interpretation.

In the following Section we introduce the concept of perfect information to characterize A. In sequel, we treat both point and confidence interval estimation problems by one sampling procedure the next sections focuse on triple sampling technique as a more credible methodology for optimal design. Finally, we discuss the sensitivity of the fixed width confidence interval, to detect shifts in the true parameter μ, in the last section.

The Cost of Perfect Information

In this section we introduce the concept of perfect information for estimating the location parameter μ of the negative exponential distribution. At this point we recall (5) and write

$$A = cn^* \, I(n^*, \sigma^2). \tag{6}$$

The term cn^* in (6) is the cost of optimal sampling, and the term $I(n^*, \sigma^2) = (n^*/\sigma^2)$ is the perfect information (in Fisher's sense) PI. By this specification we mean that the PI represents the optimal information, i.e., the amount of information required to explore a unit of variance in order to achieve the minimum expected cost (risk). Hence, A is the cost of perfect information, C_{PI}, i.e., the monetary amount that needs to be paid to explore a unit of variance in order to achieve the minimum risk. (Note that A can also be interpreted as the ratio of the optimal sampling cost relative to the optimal "estimation error" which is just PI^{-1}), Not the other way around as described by Starr and Woodroofe (1972). We emphasis that A reflects both estimation error and sampling cost.

Direct substitution of A in (6) into the loss function in (2) yields

$$L_n = cn^* I(n^*, \sigma^2)(X_{n(1)} - \mu)^2 + cn, \qquad (7)$$

And the corresponding risk in (7) becomes

$$R_n = cn^* I(n^*, \sigma^2) E(X_{n(1)} - \mu)^2 + cn. \qquad (8)$$

Now suppose that the random sample used to obtain $X_{n(1)}$ in (8) is of arbitrary size n. It follows that $E(X_{n(1)} - \mu)^2 = I^{-1}(n, \sigma^2)$, where $I(n, \sigma^2)$ is the (Fisher) sample information, SI_n. The corresponding risk in (8) can be expressed as

$$R_n = cn^*\{PI(SI_n^{-1})\} + cn = c\{n^*(RI_n) + n\}. \ (9)$$

The quantity $\{PI(SI_n^{-1})\} = RI_n$ is termed the relative information. In words, RI_n is a measure of the estimation efficiency of the sample information compared to the perfect information. When $n = n^*$, the sample information is as efficient as the perfect information since $RI_n = 1$ and $R_{n^*} = 2cn^*$, which is the optimal risk (minimum expected cost). On the other hand, when $n < n^*$, the sample information is less efficient than the perfect information since $RI_n > 1$. And when $n > n^*$, the sample information is more efficient than the perfect information since $RI_n < 1$. However, for any $n \neq n^*$, we have $R_n > R_n^*$. The apparent "anomaly" that although $R_n > R_n^*$ when $n > n^*$, the corresponding sample information is stil considered to be more efficient than the perfect information, is due to the fact that oversampling is good for estimation, per se, but bad in terms of sampling costs. In the sense that the sample information represents an estimate of the perfect information, SI_n is also termed the imperfect information.

Another useful way to compare the values of the imperfect (sample) information and the PI is through the expected cost of missed opportunity, which we express as

$$R_{MISS} = (R_n - R_{n*}) = c\{n^*(RI_n - 1) + (n - n^*)\} \quad (10)$$

Therefore, it is logically apparent that the assumption of complete knowledge of the value of A in point estimation problems is impractical. With or without knowledge of σ, unless the decision maker is "willing and able" to take a sample of infinite size, the value of A is completely unknowable (except for the value of c) within the universe of discourse of point estimation. What is being said is that any prespecification of the value of A in the context of point estimation, must involve some element of subjectivity, the consequences of which are unclear without thinking more deeply about where the methodology of point estimation fits in among more credible inferential methods based on confidence intervals.

In the next sections we elaborate further on both the specific consequences of arbitrarily specifying A in the contexts of fixed width confidence interval estimators, as well as how a knowable portion of A can be specified to satisfy practically and statistically meaningful target population-related requirements within those inferential frameworks. These achievable requirements are much less vague than the necessarily subjective (i.e., not necessarily target population-related) restrictions that are inadvertendtly imposed when the focus is limited solely to the inferential vacuum of point estimation.

Assessing the C_{PI} by Fixed Width Confidence Interval Estimation

The main goal of this section is to recast the vague universe of point estimation into a more practical universe of discourse based on objectively prespecifiable requirements. The explicit optimality criteria upon which fixed width confidence intervals are designed provide a solid foundation for statistical inference from which the cost of perfect information can be determined. This task can be successfully accomplished by connecting the perfect information for point estimation with that of confidence interval. This rationale provides the gateway to linking the C_{PI} to the characteristics of the target population, as well as the required precision, that was sorely lacking in the point estimation decision framework.

In constructing a fixed d width confidence interval for the location parameter μ of the negative exponential distribution with unknown scale parameter σ such that the coverage probability is at least a specified nominal $100(1\text{-}\alpha)\%$, it is natural to employ the interval $I_n = (X_{n(1)} - d,\ X_{n(1)})$. Satisfaction of the coverage requirement means that

$$P(\mu \in I_n) = P\{0 \leq (X_{n(1)} - \mu) \leq d\} \geq 1 - \alpha,$$

From which, we determine the corresponding optimal sample size

$$n^* = (a/d)\sigma. \tag{11}$$

Where $a = \ell n(\alpha^{-1})$ comparison of (11) with (6) implies that the corresponding perfect information is

$$PI = (a/d)\sigma^{-1} \tag{12}$$

And the corresponding cost of perfect information is

$$C_{PI} = c(a/d)^2. \tag{13}$$

We see immediately that a portion of the perfect information in (13) is, in fact, knowable. Moreover, the cost of perfect information in (13) for point estimation of the exponential location within the context of the fixed width confidence interval decision framework is completely knowable, and it reflects both the characteristics of the target population and the required precision through the constants a and d.

Now, incorporating the C_{PI} in (13) into the loss function in (2) would result in an optimal sample size in (11) serving a dual purpose. First, it provides a fixed d width confidence interval for μ with coverage probability at least the specified $100(1 - \alpha)\%$. And second, it provides a point estimator for μ with minimum expected cost.

Had σ been "known", n^* in (11) could be readily determined for use in a fixed size sampling procedure. In view of the fact that n^* and σ in (11) are both unknown, a multistage sampling procedure which simultaneously estimates n^* and σ seems appropriate. Further details on the use and properties of multistage sampling for this purpose are presented in the next section.

Triple Sampling procedure for estimating μ

As we have indicated in the previous section, both the requred optimal n^* and the scale parameter σ in (12) need to be estimated simultaneously by a multistage sampling technique. Comparing the existing techniques designed for this purpose (i.e Stein's (1945) two stage, Anscombe's (1953) and Chow and Robbins's (1965) one-by-one sequential Hall's (1983) accelerated sequential, and Hall's (1981) triple sampling) in terms of asymptotic characteristics as well as economic savings, it appears that the triple sampling procedure (sequential group sampling in three stages) is the one to employ. Thus, the triple sampling technique provides a unified sampling framework within which all the specified objectives can be achieved.

In brief, the triple sampling procedure starts with a random pilot sample X_1, $X_2,\ldots,X_m$ ($m \geqslant 2$), from wich $X_{m(1)}$ and $\hat{\sigma}_m$ are computed as initial estimates of μ and σ. The second stage sample size is determined according to the decision rule

$$N_1 = max\{m, [\gamma\ \alpha\ \hat{\sigma}_m/d] + 1\}, \qquad (14)$$

Where $0 < \gamma < 1$ is a design factor which represents the fraction of n^* to be estimated in this stage and $[\cdot]$ denotes the largest integer function (empirical results support the choice of $\gamma = 0.5$ in practice, e.g., see Hall (1981)). If the decision is to continue sampling, we combine the pilot sample with N_1-m additional observations to bring the final (third stage) sample size to

$$N = max\{N_1, [a\ \hat{\sigma}_{N_1}/d] + 1\}. \qquad (15)$$

If necessary, we observe N-N_1 further observations and terminate the sampling process. Once N is determined, we compute $X_{N(1)}$ as the point estimate for μ.

Accordingly, we construct the fixed width confidence interval,

$I_N = (X_{N(1)} - d, X_{N(1)})$, for μ.

The large sample features of the triple sample size N are given in the following Theorem) .

Theorem 1. For the triple sampling procedure (14,15) and optimal fixed n^* in (12), as $d \to 0$ we have

$$(i) E(N) = n^* - \gamma^{-1} + 0.5 + o(1)$$
$$(ii) Var(N) = \gamma^{-1} n^* + o(d^{-1})$$
$$(iii) E|N - n^*|^3 = o(d^{-2})$$
$$(iv) E(h(N)) = h(n^*) + (2\gamma)^{-1}\{(\gamma - 2)h'(n^*) + n^* h''(n^*)\} + o(d^{-2}|h''(n^*)|),$$

where $h(>0)$ is any real valued, continuously differentiable function in a neighborhood of n^* such that $\sup|h'''(n)| = O(|h'''(n^*)|)$ $n \geq m$, and prime mean derivatives.

The proof of Theorem 1 is given in Hamdy et al (1988), Hamdy *et al* (1996) and Hamdy (1997) and therefore we omit details for brevity.

(viii) We now apply Theorem 1 to evaluate the expected relative information, $E(RI_N) = PIE(SI_N^{-1})$, that measures the estimation efficiency of the triple sampling information compared to the perfect information (the expectation is used since N is a random variable). Again conditioning on N and applying Part (iv) of the theorem to evaluate $E(N^{-1})$ provides

$$E(SI_N^{-1}) = E(X_{N(1)} - \mu)^2$$
$$= I^{-1}(n^*, \sigma^2)\{1 + (4 - \gamma)/(2\gamma\ n^*)\} + o(d^2).$$

Hence the expected relative information of triple sampling is given by

$$E(RI_N) = \{1 + (4 - \gamma)/(2\gamma\ n^*)\} + o(d), \quad (16)$$

from which we see that $E(RI_N) \geq 1$. This means that the triple sampling information is less efficient than the perfect information for small n^*; however, as n^* gets large the triple sampling information will be as efficient as the perfect information. It is also clear from Part (*i*) of Theorem 1 that the expected sampling cost of the triple sampling technique is less than the optimal sampling cost. In view of (10), the relevant expected cost of missed opportunity is now defined as

$$R_{MISS} = c\{n^*(E(RI_N) - 1) + (E(N) - n^*).$$

Hence (16) and Part (i) of Theorem 1 provide $R_{MISS} = c\gamma^{-1} + o(1)$. Thus when $\gamma = 0.5$, the corresponding expected cost of missed opportunity is equivalent to the cost of sampling 2 more observations.

The following Theorem 2 extends the results in Theorem 1 to obtain the coverage probability of I_N.

Theorem 2. For the triple sampling procedure (14,15) and optimal fixed n^* in (12), as $d \to o$ the coverage probability of I_N is given by

$$P(\mu \in I_N) = 1 - \alpha - d\,\alpha\,\sigma^{-1}((\alpha - \sigma + 2)(2\gamma)^{-1}) + o(d).$$

The proof of Theorem 2 is direct application of part (*iv*) of Theorem 1, since

$$P(\mu \in I_N) = 1 - E(e^{-Nd\sigma^{-1}})$$

We have seen that a fixed width confidence interval represents our basic foundation upon which we perform inference. Thus, developing such an interval requires as much care as procedures to ensure that a credible inference will be obtained. In the following section, we discuss the ability of I_N to signify shifts in the true location parameter of the negative exponential distribution.

Sensitivity of I_N to Shifts in μ

Confidence intervals, in general, provide satisfactory information regarding the reliability of inference, see Nelson (1990, 1994) for details. It is also evident that a Confidence interval shows by its length the precision of estimation, as well as which parameter values would not be rejected if they were hypothesized as (point) null values. Therefore, careful consideration concerning the ability of the interval to detect shifts in the true parameter should also be addressed to guarantee the reliability of the inference.

To clarify this point, assume that the true parameter value is μ_0. If the parameter value has shifted to μ_1 (say), assessment of the amount of risk associated with the departure from μ_0 provides an indication concerning the ability of I_N to detect such departure. To elaborate further, we consider the following two hypotheses. The null hypothesis H0: $\mu = \mu_0$, which affirms that no departure from the true parameter has occurred, against the alternative hypothesis, H_1: $\mu = \mu_1 = \mu_0 - kd$, which asserts that the parameter value departed away from μ_0 by a distance k measured in units of d. The probability (β-risk) of not detecting such a shift in the true parameter μ_0 when, in fact, a shift has actually occurred is given by $\beta = P(\mu_0 \in I_N | H_1)$. The following Theorem 3 provides an asymptotic second order approximation of β.

Theorem 3. For the triple sampling procedure (14,15) and optimal fixed n^* in (12), and as $d \to 0$ the β-risk associated with I_N is given by

$$\beta(k) = \begin{cases} \alpha^k - \alpha^{k+1} + (2\gamma\, n^*)^{-1} \\ a\alpha^k Q(k, a, \alpha, \gamma) + o(d) \ if \ \ k \geq 0. \\ 1 - \alpha^{k+1} - (2\gamma\, n^*)^{-1} \\ a\alpha^{k+1}(k+1)(ak + a - \gamma + 2) \\ +o(d) \qquad\qquad if -1 < k \leq 0 \end{cases}$$

Where

$$Q(k, a, \alpha, \gamma) = a(1-\alpha)k^2 + (a\gamma - 2\alpha - 2a\alpha + 2)k - \alpha(2 - \gamma + a).$$

Proof of Theorem 3. First, we write

$$\beta = Ee^{-Ndk\sigma^{-1}} - Ee^{-Nd(k+1)\sigma^{-1}}$$

Next, we apply part (*iv*) of Theorem 1, for $k \geq 0$ and $-1 < k \leq 0$, then the statement of Theorem 3 is immdiate. It follows from Theorem 3 that for $k = 0$

$$\beta(0) = 1 - \alpha - d\alpha\sigma^{-1}((a - \gamma + 2)(2\gamma)^{-1}) + o(d)$$

which is actually the statement of Theorem 1.

Computation based on Theorem 3 were performed to provide a feel for the risk associated with the triple sampling confidence interval I_N. We set γ to 0.5 and $(1 - \alpha)$ was set at 0.95. We took the optimal sample size n^* to be 5, 10, 15, 20, 25, 30, 100, and 500. The departure factor varied over the range $-1 < k \leq 2.5$. We present our findings in Table(I) for some selected values of k.

Table I

$\beta - risk$

$\gamma = 0.5$, $\alpha = 0.01$

K	N*=5	N*=10	N*=15	N*=20	N*=30	N*=100	N*=500
-0.99	0.031	0.038	0.04	0.042	0.043	0.044	0.045
-0.8	0.424	0.513	0.543	0.558	0.572	0.593	0.6
-0.7	0.549	0.649	0.682	0.699	0.715	0.739	0.747
-0.6	0.646	0.744	0.776	0.793	0.809	0.832	0.84
-0.5	0.725	0.812	0.842	0.856	0.871	0.891	0.898
-0.4	0.788	0.863	0.887	0.9	0.912	0.929	0.935
-0.3	0.839	0.9	0.92	0.93	0.94	0.954	0.959
-0.2	0.879	0.927	0.943	0.951	0.959	0.97	0.974
-0.1	0.91	0.947	0.959	0.966	0.971	0.98	0.983
0	0.934	0.962	0.971	0.976	0.981	0.987	0.989
0.2	0.541	0.467	0.443	0.431	0.419	0.401	0.396
0.4	0.336	0.246	0.217	0.202	0.187	0.166	0.159
0.6	0.203	0.133	0.109	0.098	0.086	0.069	0.064
0.8	0.117	0.071	0.055	0.048	0.04	0.029	0.026
1	0.064	0.037	0.028	0.023	0.019	0.013	0.01
1.2	0.034	0.019	0.014	0.011	0.009	0.005	0.004
1.4	0.017	0.009	0.007	0.006	0.004	0.002	0.002
1.6	0.009	0.005	0.003	0.003	0.002	0.001	0.001
2	0.002	0.001	0.001	0.001	0	0	0
2.3	0.001	0	0	0	0	0	0

Table I discloses several points regarding the performance of the triple sampling fixed width confidence interval. We notice that β increases as n^* increases over the range $-1 < k \leq 0$. It is also clear that β increases rapidly when n^* is small (≤ 30) and slows down for larger values of n^*. For specific choice of n^* and for $-1 < k \leq 0$, we observed that β increases as k increases to attain its maximum value $(1 - \alpha)$ when $k = 0$. On the contrary, for $0 \leq k \leq 2.5$, β decreases as k increases and approaches 0 as k approaches (2.5). The above discussion indicates that I_N is unable to detect shifts in the true parameter in general and is very insensitive to left shifts in particular. Therefore, careful consideration to the insensitivity of the interval should also be considered when making inference about μ.

Dicussion

In summary, we have shown that a fixed width confidence interval based on a triple sampling scheme can serve as a credible building block for point estimation about the exponential location paramter. The perfect information obtained from the specifiable fixed width confidence interval requirements that can be incorporated into the loss function in (2) yields a reliable point estimate of the mean with nearly the corresponding minimum risk. Specifically, the absolute difference between $X_{N(1)}$ and μ is guaranteed with 100(1-α)% confidence to be at most d and the expected cost of missed opportunity (or "the risk of not knowing the variance" (e.g., see Simons, 1968) for point estimation is $c\gamma - 1 + o(1)$. Once again, this is equivalent to the cost of sampling 2 more observations when the design factor $\gamma = 0.5$. We have also pointed out that the sensitivity of the interval should be addressed to ensure the reliability of our statement regarding the location paramter.

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الملخص

تكلفة المعلومات التامة
لتقدير معلمة الموضع للتوزيع الأسي

حسني إبراهيم حمدي
جامعة الكويت

تستعرض هذه الدراسة دور تكلفة المعلومات التامة لتقدير معلمة الموضع للتوزيع الأسي. لقد تم دمج طرق التقدير بالنقطة وطرق التقدير باستخدام فترات ثقة في طريقة واحدة باستخدام تكلفة المعلومات التامة للاستفادة من المعلومات المتاحة بطريقة مثلى، وبالتحديد فقد تم مراجعة طريقة التقدير بالنقطة من خلال إطار أشمل وأوسع وهو إطار فترات الثقة حيث أن المعلومات اللازمة لإجراء التقدير بالنقطة هي معلومات غير معروفة مسبقاً. ولقد تم تقديم هذه الطريقة من خلال التجربة الثلاثية. أيضاً تم التعرف على بعض المؤشرات المتعلقة بحساسية التجربة الثلاثية في التعرف على التغيرات التي قد تطرأ على معلمة التوزيع الأسي.

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مجلة دراسات الخليج والجزيرة العربية

تصدر عن جامعة الكويت ـ مجلس النشر العلمي

رئيس التحرير :

أ. د. أمل يوسف العذبي الصباح

مجلة علمية فصلية محكمة تصدر أربعة أعداد في السنة ، بالإضافة إلى اصدارات خاصة في المناسبات .

● صدر العدد الأول منها في يناير ١٩٧٥ .

● تعنى المجلة بنشر :

ـ البحوث والدراسات المتعلقة بشئون منطقة الخليج والجزيرة العربية السياسية والاقتصادية والاجتماعية والثقافية والعلمية .... الخ .

● مراجعات الكتب العربية والأجنبية المهتمة بمنطقة الخليج والجزيرة العربية .

ـ تقارير عن أهم الندوات التي تعقد في داخل الكويت وخارجها بالإضافة إلى البيبلوجرافيا بالعربية والانجليزية .

● صدر عن المجلة :

أ ـ مجموعة من المنشورات المتخصصة .

ب ـ مجموعة من الاصدارات الخاصة المتعلقة بمنطقة الخليج والجزيرة العربية .

ج ـ سلسلة كتب وثائق الخليج والجزيرة العربية من ١٩٧٥ ـ ١٩٨٢ .

د ـ عقد الندوات التي تهم المنطقة أو المساهمة فيها واصدارها في كتب .

الاشتراك السنوي :

أ ـ داخل الكويت : ٣ د .ك للأفراد ـ ١٥ د .ك . للمؤسسات .

ب ـ الدول العربية : ٤ د .ك . للأفراد ـ ١٥ د .ك . للمؤسسات .

ج ـ الدول الأجنبية : ١٥ دولار للأفراد ـ ٦٠ دولار للمؤسسات .

المقـــر :

جامعة الكويت ـ الشويخ

مبنى مجلس النشر العلمي

هـاتف : ٤٨٣٣٧٠٥

٤٨٣٣٢١٥

بـدالة : ٤٨٤٦٨٤٣/ ٤٠٦٦

٤٨٤١٥٣٨/ ٤٠٦٧

فاكس : ٤٨٤٥٣٧٢

جميع المراسلات توجه باسم رئيس التحرير على العنوان الآتي :

ص .ب . : ١٣٠٧٣ ـ الخالدية ـ الكويت ـ الرمز البريدي ٧٢٤٥١