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HEDGING AND ARBITRAGE WHEN THE BASE CURRENCY IS PEGGED TO A BASKET

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This paper examines hedging and arbitrage in the foreign exchange market, in the particular case of the base currency being pegged to a basket. The sources of foreign exchangs risk arising from taking long and short positions on a multi-currency portfolio are identifed, and the findings are used to design some hedging and arbitrage rules. These rules make it possible to minimise foreign exchange risk arising from a long position on a portfolio, and to arbitrage the interest rate differential by taking either short or long positions on the portfolio. Some empirical evidence on the hedging rules is presented using the SDR as a base currency.

In this paper, an attempt is made to identify the sources of risk arising from taking long and short positions on multi-currency portfolios when the base currency is pegged to a basket. Once the sources of risk are identifed, it will be possible to design some rules for hedging and arbitrage by taking a short position on the base currency and a long position on a multi-currency portfolio or vice versa. For this purpose, hedging is defined as covering (fully or partially) the foreign exchange risk arising from taking a long position on a multi-currency portfolio. Thus, the objective is to determine the weights of the currencies in the portfolio in such a way as to minimise foreign exchange risk, given that the currencies are chosen a priori on the basis of various (subjective) criteria. On the other hand, arbitrage involves taking short and long positions in such a way as to utilise the interest rate differentials while "eliminating" foreign exchange risk.

Although there seems to be some overlapping in the two concepts, the diference is the following. In the case of hedging, it is assumed that the investor chooses to invest in foreign currencies for other reasons than to maximise the differential return (e.g. a temporary flight of capital seeking safe haven while

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bearing in mind the possibility of eventual repatriation of funds).[1] In this case, the investor must be concerned about foreign exchange risk which he wishes to minimise, subject to the constraints implied by the portfolio composition. On the other hand , the objective of arbitrage is to construct the foreign currency portfolio (including the choice of currencies) in such a way as to eliminate (as opposed to minimise) foreign exchange risk.[2] In the case of arbitrage, the foreign currency portfolio may therefore be either a borrowing portfolio (short position) or an investment portfolio (long position), depending on the interest rate configuration.

As a starting point it seems useful to point out what is meant by a 'base currency' and a 'currency that is pegged to a basket'. A base currency may be the local currency of the international investor, the currency in which the financial statements of the investor are expressed, or the currency in which the shareholders of a mutual fund pay their contributions and receive their dividends But in general, a base currency is a currency in which performance is measured, such that the foreign exchange gain (loss) results from the appreciation (depreciation) of other currencies against it. A currency that is pegged to a basket is a currency whose exchange rate against a numeraire (normally the U.S dollar) is measured according to a formula that reflects the composition of the basket of currencies to which it is pegged.[3]

The rest of this paper is organised as follows. The starting point is a brief description of the concept of foreign exchange risk. This is followed by a presentation of the procedure used to measure foreign exchange return and to identify the sources of this return and, therefore, the sources of foreign exchange risk. On the basis of these findings, the hedging and arbitrage rules are then presented. This is followed by a presentation of some empirical evidence on the hedging rules using the SDR as a base currency. Finally, we end up with some concluding remarks.

The Concept of Foreign Exchange Risk

In investment analysis, the term 'foreign exchange risk' refers to the variability of overall return resulting purely from (unanticupated) changes in exchange rates

1 One of the good examples in this respect is the Kuwaiti dinar (KD) which is pegged to a basket of undeclared components. Temporar capital flight from Kuwait has been quite common (e.g. Moosa, 1983, 1989).

2 This definition of arbitrage is consistent with the description of the concept in the finance literature, that it is a risk-free operation designed to exploit some anomalies in financial markets, Of course, arbitrage as considered here is not completely risk free, since it is assumed that the objective of arbitrage is to make the operation as close as possible to being risk free. This case is indeed similar to the case of uncovered interest arbitrage when the underlying currencies have stable exchange rates. In the extreme case of fixed exchange rates, there is obviously no foreign exchange risk (except the risk of realignment).

3 See Argy (1990) and Flood et al. (1989) for a description of exchange rate arrangements.

when the investment is not denominated in the base currency. The advent of flexible exchange rates following the collapse of the Bretton Woods System in 1971 has created another source of financial risk associated with the activity of investing across national frontiers. The result is that the advantages gained from international diverisification may be realised only by bearing foreign exchange risk.

Recent years have witnessed ever increasing volatility of exchange rates. Volatility is defined by Frenkel and Goldstein (1988) as "short-term fluctuations of nominal or real exchange rates about their long-term trends". In this paper we are concerned with nominal exchange rates, which have been shown by Frenkel and Mussa (1980) to be more volatile than price levels, implying that the variability of real exchange rates mainly reflects the variability of nominal rates. Frenkel and Goldstein (1988) point out that volatility has increased by a factor of five since floating was adopted. Moreover, exchange rates have been unpredictable as demonstrated by Mussa (1983) by means of market indicators (interest rate differentials and forward discounts), and by Frankel and Froot (1987) by survey data on exchange rate expectations .

More often than never, changes in exchange rates tend to surpass in importance (in terms of contribution to overall return) the nominal return in foreign currency terms. As a result, changes in exchange rates have become a singinficant factor contributing to the overall riskiness of investment in multi-currency portfolios. Indeed , the total return on a multi-currency portfolio is more volatile under floating exchange rates than under fixed exchange rates unless the return in foreign currency terms is negatively correlated with changes in exchange rates, which is not necessarily the case.

The Measurement of Foreign Exchange Return

Nominal return expressed in base curency, R_b, consists of two components: pure nominal return,R_f, expressed in foreign currency terms, and the foreign exchange return, F. In a two-period two-currency model, the following relationship can be specified

$$R_b = R_f + F(1+R_f) \tag{1}$$

where $F= \Delta X/X$, and X is the exchange rate expressed as units of the base currency per one unit of the foreign currency. This means that a higher (lower) X indicates appreciation (depreciation) of the foreign currency against the base currency, and hence foreign exchange gain (loss)[4]. If an investment is made in a multi-currency portfolio containing m currencies, the return resulting from changes in exchange rates is a weighted average of the individual currency returns, the weights being the percentages of currencies in the portfolio. This is given by

4 Foreign exchange gain or loss is in general called foreign exchange return.

$$F = \sum_{i=1}^{m} \beta_i F_i \tag{2}$$

where F_i is the return resulting from the change in the exchange rate X_i. This analysis deals with continuous time, in which case foreign exchange return is given by

$$F_i = \frac{1}{X_i}\frac{dX_i}{dt} = \hat{X}_i \tag{3}$$

Thus, for a multi-currency portfolio, foreign exchange return is given by

$$F = \sum_{i=1}^{m} \beta_i \hat{X}_i \tag{4}$$

which implies that it is a weighted average of the changes in exchange rates. we will now present a theoretical model which is used to identify the sources of foreign exchange risk arising from a multi-currency portfolio. The model is based on a set of integers given by

$$I = \{1,2,\ldots,m,m+1,\ldots,n,n+1,n+2,\ldots,k\} \tag{5}$$

representing a universe of currencies which can be chosen to be in the basket and in the portfolio, excluding the numeraire which is the U.S. dollar. The currencies of which the basket is composed are represented by C_i, where i belongs to a subset of I currencies such that

$$\{i \in I : i \leq n\} \tag{6}$$

This means that apart from the numeraire, C_0, there are n currencies in the basket such that i= 1.2,...,n. Mathematically, the basket can be represented by an equation expressing the relationship between the exchange rate of the pegged (base) currency per dollar, S, and the exchange rates per dollar of the curencies the basket is composed of. For the purpose of this model, the following relationship is specified

$$S = \alpha_0 E_1^{\alpha_1} E_2^{\alpha_2} \ldots E_n^{\alpha_n} = \alpha_0 \prod_{i=1}^{n} E_i^{\alpha_i} \tag{7}$$

where E_i is the exchange rate of currency i against the dollar and a_i, is a measure of the weight of currency i in the basket. Since the base currency is not the dollar, the need arises (for the purpose of deriving an expression for F) to express the cross exchange rates, e_i, in terms of the bilateral exchange rates against the dollar. This relationship is given by

$$e_i = \frac{S}{E_i} \tag{8}$$

implying that

$$\hat{e}_i = \hat{S} - \hat{E}_i \tag{9}$$

Equation (7) can be written in a logarithmic form as

$$\log S = \log \alpha_0 + \sum_{i=1}^{n} \alpha_i \log E_i \tag{10}$$

Differentiating with respect to time yields

$$\frac{1}{S}\frac{dS}{dt} = \sum_{i=1}^{n} \alpha_i \frac{1}{E_i}\frac{dE_i}{dt} \tag{11}$$

which can be rewritten using the notation as

$$\hat{S} = \sum_{i=1}^{n} \alpha_i \hat{E}_i \tag{12}$$

We are now in a position to derive the equations representing foreign exchange return, F. Obviously, the specification of the equation depends on the composition of the portfolio, and for this purpose four cases are considered. Table 1 summarises the cases using set theory notation.

Table1

Currency Composition of Basket and Portfolio

Case	Currencies in Basket	Currencies in Portfolio
1	$\{i \in I : i \leq n\}$	$\{i \in I : i \leq n\}$
2	$\{i \in I : i \leq n\}$	$\{i \in I : i \leq m\}$
3	$\{i \in I : i \leq n\}$	$\{i \in I : i \leq k\}$
4	$\{i \in I : i \leq n\}$	$\{i \in I : i \leq m \text{ or } n < i \leq k\}$

In general, foreign exchange return depends on changes in the underlying exchange rates. Thus:

$$F = \mathbf{B}'\hat{\mathbf{X}} \tag{13}$$

where

$$\mathbf{B} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_q \end{bmatrix}, \qquad \hat{\mathbf{X}} = \begin{bmatrix} \hat{S} \\ \hat{e}_1 \\ \vdots \\ \hat{e}_q \end{bmatrix}$$

and q = n, m, k depending on the composition of the portfolio as shown in Table 1. The derivation of the expression for F is shown in the appendix, while the results are summarised in Table 2.

Table 2

Sources of Foreign Exchange Return and Risk

Case	Number of Currencies in Basket and Portfolio*	Currencies in Basket and Portfolio	Currencies in Basket Only	Currencies in Portfolio Only
1	n,n	$\sum_{i=1}^{n}(\alpha_i-\beta_i)\hat{E}_i$		
2	n,m	$\sum_{i=1}^{m}(\alpha_i-\beta_i)\hat{E}_i$	$\sum_{i=m+1}^{n}\alpha_i\hat{E}_i$	
3	n,k	$\sum_{i=1}^{n}(\alpha_i-\beta_i)\hat{E}_i$		$-\sum_{i=n+1}^{k}\beta_i\hat{E}_i$
4	$n,m+k-n$	$\sum_{i=1}^{m}(\alpha_i-\beta_i)\hat{E}_i$	$\sum_{i=m+1}^{n}\alpha_i\hat{E}_i$	$-\sum_{i=n+1}^{k}\beta_i\hat{E}_i$

*The number excludes the numeraire (U.S. dollar) which is taken to be currency Zero. a_i, is the weight of currency i in the basket, β_i , is the weight of the currency in the portfolio, and $\hat{E}_i$ is the rate of change of its exchange rate against the dollar.

Hedging and Arbitrage Rules

Hedging Rules

The objective of the hedging rules is that given the selection of currencies to be included in the basket, the weights of these currencies must be chosen in such a way as to minimise foreign exchange risk. It must be made clear that the word 'minimise' as used here does not imply the reduction of the foreign exchange risk (as measured by the variance of the foreign exchange return) in terms of a constrained optimisation problem derived, for example, from a mean-variance model. It rather implies the choice of a unique set of weights that will produce a smaller degree of risk than any other alternative set of weights. It must also be emphasised that the choice of the currencies (as opposed to their weights) is determined a priori for reasons other than the reduction of risk. Thus, the hedging process only involves the choice of weights of pre-selected currencies. The following are the hedging rules applicable to each of the four cases under consideration .

In case I foreign exchange return (and hence risk) arises from currencies in the basket and in the portfolio (Table 2). It is obvious that F=0 if α_i=β_i for i= 1,2,...n. This means that foreign exchange return can be reduced to zero, eliminating foreign exchange risk, by constructing the portfolio in such a way as to equate the percentage of each currency with its weight in the basket.[5] 'This rule allows the investor to obtain a rate of return equal to that in foreign currency, R f, without bearing foreign exchange risk. If we consider an h- period model in which t=1,2,...h then it is obvious that if the condition $\alpha_i = \beta_i$ is satisfied it follows that F_i = 0 for t= 1,2,...h, and so the variance of F_i will be zero, indicating no foreign exchange risk. If on the other hand $\alpha_i \neq \beta i$ then $F_i \neq 0$ and the variance will not be zero, i.e. foreign exchange risk will be present.

In case 2, the foreign exchange return consists of two components. The first originates from currencies included in the basket and in the portfolio, while the second is due to currencies included in the basket only(Table 2). In this case F$\neq$ 0 when α_i=β_i for i=1,2,...m. Hence, it is not possible to eliminate foreign exchange risk completely, but it can be minimised by setting α_i=β_i for i=1,2,...m. The size of the residual risk is then determined by the weights allocated to currencies other than the dollar. Obviously, if the whole of the remaining weight is assigned to the dollar, foreign exchange risk would be eliminated completely, and this case will be identical to case 1.

In case 3 ,there is also a second component of foreign exchange return derived from currencies included in the portfolio only (Table 2). Again, $F \neq 0$ when $\alpha_i = \beta_i$ for i- 1,2,...n. However, setting α_i=β_i for i = 1,2,...n will minimise foreign exchange risk. The size of the residual risk will also be a function of the weights assigned to currencies other than the dollar.

In case 4, there are three components of foreign exchange return. Setting α_i = β_i for i = 1,2,...m will not eliminate foreign exchange risk because of the effect of currencies in the basket only and those in the portfolio only. It is possible, however, that these effects may cancel out each other since they work in opposite directions.

Arbitrage Rules

The objective of arbitrage is to utilise the interest rate differential while eliminating foreign exchange risk. For simplicity, we will ignore the bid-offer spread on interest rates and assume that the borrowing and lending rates for each currency are equal. Let the interest rate on the base currency be r_h, and on foreign currency, C_i, be r_i(i = 1,2,...,k). Thus, the effective interest rate on the foreign

5 If the basket is unknown (not revealed), it is possible to find out its composition by estimating equation (10). To take account of the possibility of changes in the weights, a rolling regression will be superior to OLS.

currency portfolio is $\sum_{j}^{k}\beta r_{i}$. Let us also start with the general case where there are currencies that are included in the basket and in the porfolio, others that are included in the basket only, and those included in the portfolio only. If a short (long) position is taken on the base currency while a long (short) position is taken on the portfolio, then the gain or loss, π , will be given by

$$\pi=\sum_{i=1}^{k}\beta_i r_i - r_b + \sum_{i=1}^{m}(\alpha_i-\beta_i)\hat{E}_i \div \sum_{i=m+1}^{n}\alpha_i\hat{E}_i - \sum_{i=n+1}^{k}\beta_i\hat{E}_i \tag{14}$$

If the objective is to eliminate foreign exchange risk while gaining the interest rate differential, then the following conditions must be satisfied: $k = n$ such that $Ci = C_j$, and $\alpha i = \beta i$ for i = 1,2,...m.[6] If these conditions are satisfied, equation (14) reduces to

$$\pi=\sum_{i=1}^{k}\beta_i r_i - r_b \tag{15}$$

because

$$\sum_{i=1}^{m}(\alpha_i-\beta_i)\hat{E}_i + \sum_{i=m+1}^{m}\alpha_i\hat{E}_i = \sum_{i=1}^{m}(\alpha_i-\beta_i)\hat{E}_i = 0 \tag{16}$$

and

$$\sum_{i=n+1}^{k}\beta_i\hat{E}_i = 0 \tag{17}$$

Equation (15) obviously indicates the possibility of obtaining risk-free profit because all interest rates are known in advance. It also implies that there is no possibility for arbitrage if

$$r_b = \sum_{i=1}^{n}\beta_i r_i \tag{18}$$

which means that if the interest rate on the base (pegged) currency is calculated as a weighted average of the interest rates on the components of the

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6 These two conditions imply the following: (i) the currencies in the portfolio are equal in number and identical to those in the basket, and (ii) the weight assigned to a particular currency in the portfolio is equal to its weight in the basket.

basket, arbitrage will not be profitable.[7] If however, condition (18) is not satisfied, then the following can be done. If $r_b < \Sigma\ \beta_i r_i$ a short position should be taken on the base currency and a long position on the portfolio. If, on the other hand, $r_b >$ $\Sigma \beta_i r_i$ then the opposite position should be taken.[8]

Empirical Results

In this section, some empirical evidence on the hedging rules is presented using the SDR as a base currency. The reason why the SDR is chosen is that the underlying basket of currrencies is known with certainty, in which case we do not have to worry about measurement errors. If therefore the results turned out to be unfavourable, we can conclude that the hedging methods presented in this paper do not work well rather than to attribute the outcome to measurement errors.

The empirical evidence presented here is based on ex-post simulations using hypothetical portfolios. The simulations are based on monthly data covering the period 1986-87. The reason why this period is chosen is to show that these rules do work even during periods of highly volatile exchange rates.

We first test case 1 in which currencies in the basket are the same as currencies in the portfolio. Four different portfolios are constructed, as shown Table 3, and for each portfolio the foreign exchange return is calculated on a monthly basis over the estimation period. The basket weights are used as percentages of currencies in portfolio A, while arbitrary weights are used in portfolios B, C and D. The Table also shows the means and standard deviations of the 24 monthly observations of the calculated foreign exchange return. Finally, the Table reports the t statistics for testing the null hypothesis $\mu = 0$ against the alternative $\mu \neq 0$ where u is the mean foreign exchange return.

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7 This kind of arbitrage is not proftable when the base currency is a composite currency like the ECU and the SDR because the interest rate on these currencies is calculated on the basis of equation (18). However, there are other pegged currencies (such as the KD), for which equation (18) is not satisfied, and thus arbitrage is profitable. In the case of the KD, however, the components of the basket are not revealed, and thus the ai's are not known in advance in order to satisfy the condition $\alpha i = \beta i$. What can be done here is to estimate the αi's empirically. But this means that there is some risk arising from the estimation errors and from the possibility of policy changes in the αi's. So, there is no "free lunch" after all, but it seems that this kind of risk is well worh taking.

8 If there is a difference between borrowing and lending rates, as is normally the case in practice, borrowing rates will be used for the currencies on which a short (long) position is taken.

Table 3

Portfolio Composition and Foreign Exchange Return (Case 1)

	A	B	C	D
Portfolio Composition (%)				
Dollar	42.0	20.0	90.0	10.0
Yen	17.8	20.0	2.0	22.5
D M	13.9	20.0	2.0	22.5
STG	10.9	20.0	2.0	22.5
FF	15.4	20.0	2.0	22.5
Foreign Exchange Return				
Mean	0.03	0.35	-0.80	0.51
Standard Deviation	0.12	0.70	1.48	0.99
t (df = 23)	1.06	2.45	-2.63	2.51

The objective of this empirical exercise is to show that foreign exchange return on a portfolio constructed such that $\alpha i = \beta i$ (portfolio A) is nearly zero and has the lowest standard deviation as compared with the other portfolios. The t statistics confirm the acceptance of the null hypothesis for portfolio A and its rejection for portfolios B, C and D. In all of these portfolios the mean foreign exchange return is different from zero at the 5% significance level. Moreover, the standard deviation of the foreign exchange return for portfolio A is indeed lower than those of the other portfolios. Figure 1 illustrates the findings graphically, showing monthly foreign exchange returns for the four portfolios over the estimation period. Foreign exchange return may also be calculated for longer periods. Table 4 shows the results for three periods, the 1st of which is outside the estimation period. These results confirm earlier findings.

Table 4

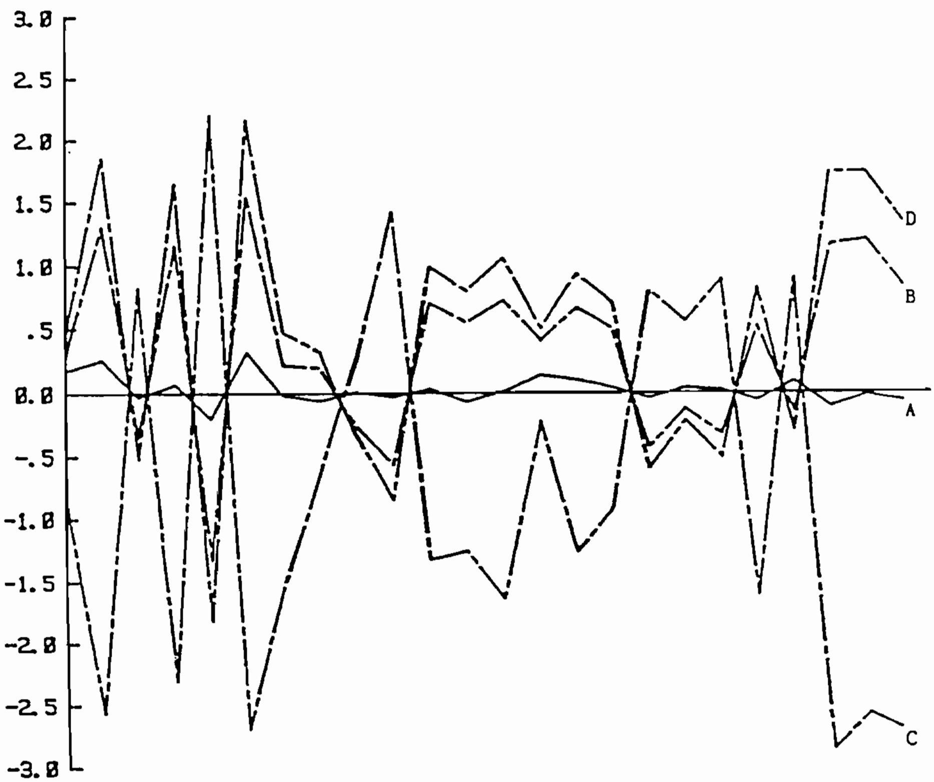
Foreign Exchange Returns over Long Periods (Case 1)

	A	B	C	D
DEC 85-DEC 86	0.71	3.68	-8.15	5.35
DEC 86-DEC 87	0.64	5.37	-10.07	7.55
DEC 87-DEC 88	0.29	-2.34	6.10	-3.53

Figure 1

Monthly Foreign Exchange Return (1986 - 87)

(Case 1)



It is important to note that foreign exchange return on portfolio A in Table 5 is not identically equal to zero as predicted by the theoretical model. One reason for that is that the theoretical model is derived in continuous time while empirical testing is based on discrete time. However, if we have a large sample of annual observations on foreign exchange return, we will be able to conduct the t test of the mean as in the case of monthly observations. In this case, the t test should again verify that the mean value of foreign exchange return is not significantly different from zero.

The socond set of empirical results pertains to cases 2,3 and 4. Table 5 shows the mean foreign exchange return over the period 1986-87 measured in total, the component arising from currencies in basket, and the component arising from currencies in portfolio. The t statistics are calculated to test the significance of the difference between mean total return and that from the basket and /or portfolio.

Table 5

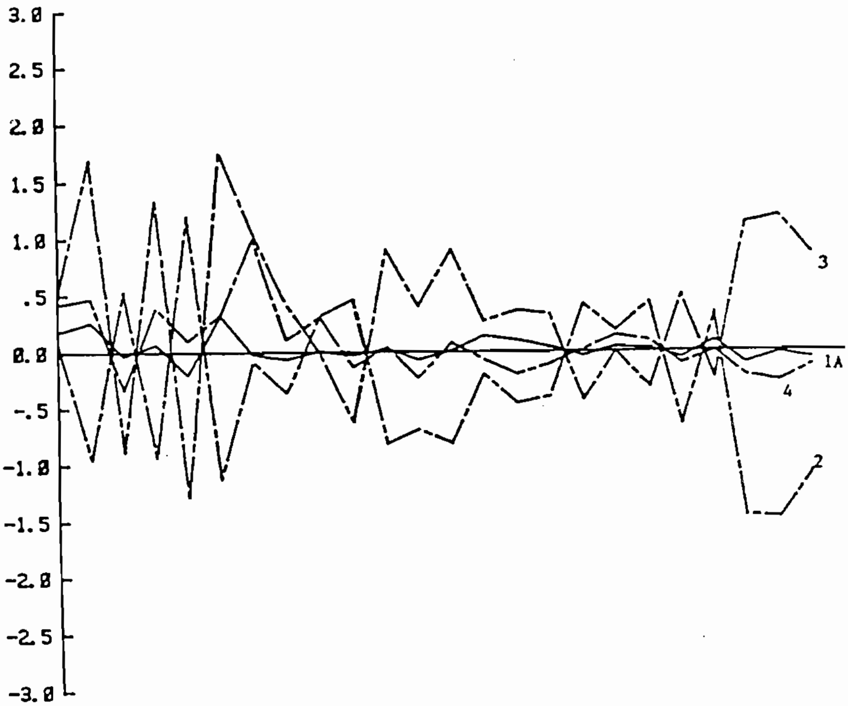
Portfolio Composition and Foreign Exchange Return (Cases 2, 3 and 4)

	2	3	4
Portfolio Composition			
DOllar	68.3	22.0	48.3
Yen	17.8	17.8	17.8
DM	13.9	13.9	13.9
STG	0	10.9	0
FF	0	15.4	0
SF	0	10.0	10.0
DFL	0	10.0	10.0
Foreign Exchange Return			
Total	-0.31	0.41	0.07
Basket	-0.32	-	-0.32
Portfolio	-	0.37	0.37
Standard Deviation	0.69	0.77	0.29
Statistics (Mean Difference)	-0.59	1.71	1.15

Figure 2

Monthly Foreign Exchange Return (1986 - 87)

(Case 1A, 2,3 and 4)



The results presented in Table 5 lead to two conclusions. The first is that cases 2,3 and 4 produce higher and more volatile foreign exchange return than the basic case (portfolio A of case 1) which satisfies the allocation condition $\alpha i = \beta i$. This is also shown by Figure 2. The second conclusion is that since in all of these cases (2, 3 and 4) the condition $\alpha i = \beta i$ is satisfied for currencies included both in the basket and in the portfolio, foreign exchange return arising from these currencies should be zero. Therefore, total foreign exchange return should, on average, be equal to that arising from the basket and /or the portfolio. The t statistics of the mean difference are calculated to test the null hypothesis

$$\mu_t = \mu_b + \mu_p \quad (19)$$

against the alternative hypothesis

$$\mu_t = \mu_b + \mu_p \quad (20)$$

where μ_t is the mean total return, μ_b is mean return arising from currencies in the basket, and μ_p is mean return arising from currencies in the portfolio. The test statistics lead to the acceptance of the null hypothesis. Figures 3,4 and 5 show the behaviour of the foreign exchange return in the three cases. The same results hold for longer periods as shown in Table 6.

Table 6

Foreign Exchange Return over Long Periods

	2	3	4
Dec 85-Dec 86			
Total	-2.00	5.63	2.92
Basket	-2.59	-	-2.59
Portfolio	-	4.28	4.28
Dec 86-Dec 87			
Total	-4.59	-4.98	-0.25
Basket	-4.83	-	-4.83
Portfolio	-	3.95	3.95
Dec 87-Dec 88			
Total	3.53	-2.82	0.42
Basket	3.43	-	3.43
Portfolio	-	-3.40	-3.40

Figure3

Monthly Foreign Exchange Return (1986 - 87)

(Case 2)

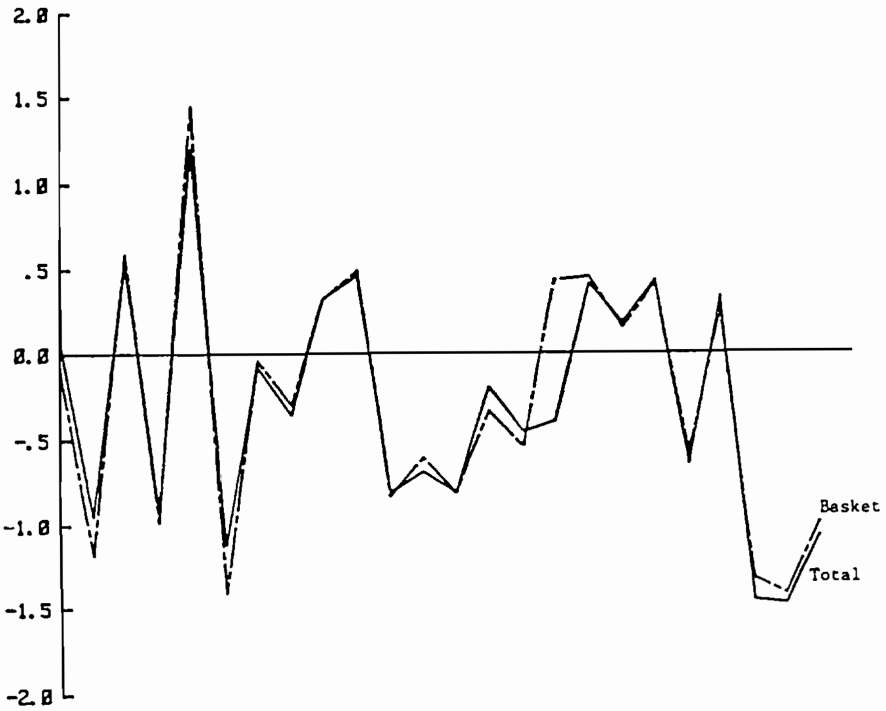


Figure4

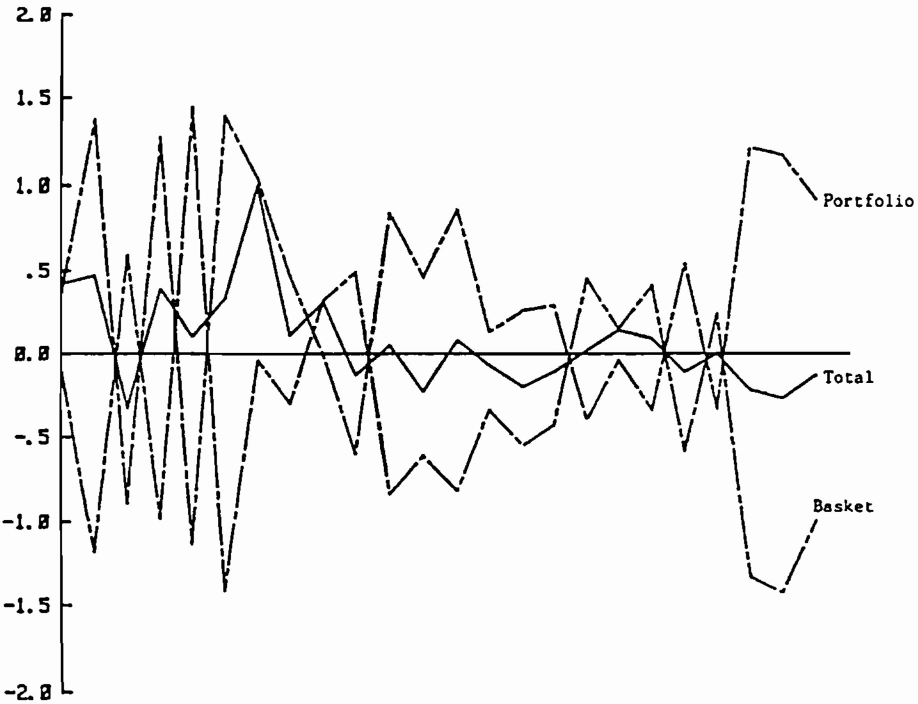
Monthly Foreign Exchange Return (1986 - 87)

(Case 3)

Figure5

Monthly Foreign Exchange Return (1986 - 87)

(Case 4)



Obviously, the SDR cannot be used as a base currency to provide empirical evidence on the arbitrage rules, because in this case the condition implied by equation (18) is not violated. For this purpose we have to use a currency like the KD by starting with the estimation of the basket components. The problem with the KD, however, is that there seems to be some evidence that the Central Bank of Kuwait has not been very consistent in adhering to the basket (see, for example, Moosa, 1989). However, this was definitely not the case in the first half of the 1980s. Confirming or refuting this proposition is a task that I will leave for future research.

Conclusion

This paper attempted to develop some rules for hedging and arbitrage using multi-currency portfolios when the base currency is pegged to a basket: rules were designed to deal with two situations: (i) when the objective is to minimise foreign exchange risk if the currencies which make up the investment portfolio are selected a priori by considerations other than the rate of return; and (ii) when arbitraging the interest rate differerential between the base currency and a portfolio of foreign currencies. In the first situation, the currencies included in the portfolio and those included in the basket (apart from the numeraire) should be given the same weight in the portfolio as in the basket. In the second situation, arbitrage is profitable if a short position is taken on the base currency and a long position is taken on the portfolio (or vice versa), provided that the portfolio is identical in composition to the basket. It seems to be the case that it is worthwhile to try these rules in practice.

Appendix

Representation of the Foreign Exchange Return

Case 1

In this case, the currencies included in the portfolio are identical to those included in the basket such that i = 1,2,..n. Foreign exchange return arises from two sources: (i) changes in the exchange rates of the base currency against the numeraire (the U.S dollar), and (ii) changes in the exchange rates of the base currency against the other n currencies. Assuming that the portfolio is composed of β_0 dollar and β_i other currencies such that

$$\sum_{i=0}^{n} \beta_i = 1 \tag{A.1}$$

Hence

$$F = \beta_0 \hat{S} + \beta_1 \hat{e}_1 + \beta_2 \hat{e}_2 + \cdots + \beta_n \hat{e}_n$$

$$= \beta_0 \hat{S} + \beta_1 (\hat{S} - E_1) + \beta_2 (\hat{S} - E_2) + \cdots + \beta_n (\hat{S} - E_n)$$

or

$$F = \hat{S}\sum_{i=0}^{n}\beta_i - \sum_{i=1}^{n}\beta_i\hat{E}_i \qquad \text{(A.2)}$$

Combining (A.2) with (A.1) and (12), we obtain

$$F = \sum_{i=1}^{n}(\alpha_i - \beta_i)\hat{E}_i \qquad \text{(A.3)}$$

Case 2

In this case, the number of currencies included in the portfolio, m, is less than that of currencies included in the basket, n, such that *i = 1,2,.., m,m + 1,m+2,..,n. Therefore*

$$F = \beta_0\hat{S} + \beta_1\hat{e}_1 + \beta_2\hat{e}_2 + \cdots + \beta_m\hat{e}_m$$

$$= \beta_0\hat{S} + \beta_1(\hat{S} - E_1) + \beta_2(\hat{S} - E_2) + \cdots + \beta_m(\hat{S} - E_m)$$

or (A.4)

$$F = \hat{S}(\beta_0 + \beta_1 + \cdots + \beta_m) - (\beta_1\hat{E}_1 + \beta_2\hat{E}_2 + \cdots + \beta_m\hat{E}_m)$$

Combining (A.4) With (A.1) and (12). we obtain

$$F = \sum_{i=1}^{n}\alpha_i\hat{E}_i - \sum_{i=1}^{m}\beta_i\hat{E}_i \qquad \text{(A.5)}$$

Since

$$\sum_{i=1}^{n}\alpha_i\hat{E}_i = \sum_{i=1}^{m}\alpha_i\hat{E}_i + \sum_{i=m+1}^{n}\alpha_i\hat{E}_i \qquad \text{(A.6)}$$

It follows that

$$F = \sum_{i=1}^{m}(\alpha_i - \beta_i)\hat{E}_i + \sum_{i=m+1}^{n}\alpha_i\hat{E}_i \qquad \text{(A.7)}$$

Case 3

IN this case the number of currencies included in the protfolio, *k*, is larger than that in the basket, *n* such that *i = 1,2,... n , n 1, n +2,... k*. Therefore

$$F=\beta_0\hat{S}+\beta_1\hat{e}_1+\beta_2\hat{e}_2+\cdots+\beta_k\hat{e}_k$$

or

$$F=\hat{S}-\sum_{i=1}^{k}\beta_i\hat{E}_i \tag{A.8}$$

Since

$$\sum_{i=1}^{k}\beta_i\hat{E}_i=\sum_{i=1}^{n}\beta_i\hat{E}_i+\sum_{i=n+1}^{k}\beta_i\hat{E}_i \tag{A.9}$$

it follows that

$$F=\sum_{i=1}^{n}\alpha_i\hat{E}_i-\left[\sum_{i=1}^{n}\beta_i\hat{E}_i+\sum_{i=n+1}^{k}\beta_i\hat{E}_i\right]$$

or

$$F=\sum_{i=1}^{n}(\alpha_i-\beta_i)\hat{E}_i+\sum_{i=n+1}^{k}\beta_i\hat{E}_i \tag{A.10}$$

Case 4

In this case, the portfolio is composed of some currencies that are included in basket and others that are not, such that i = 1,2,..m,m +1,.. n,n+1,n + 2,... k. Currencies included in the basket and the portfolio are *1,2,* ..., m, currencies included in the basket but not in the portfolio are m +1.m + 2,... n; and those included in the *portfolio* n + 1, n + 2 , ... K. Thus, the total number of currencies in the *portfolio* is m + k - n. Hence

$$F=\beta_0\hat{S}+\beta_1\hat{e}_1+\beta_2\hat{e}_2+\cdots\beta_m\hat{e}_m+\beta_{n+1}\hat{e}_{n+1}+\beta_{n+2}\hat{e}_{n+2}+\cdots+\beta_k\hat{e}_k$$

or

$$\begin{aligned}F=&\beta_0\hat{S}+\beta_1(\hat{S}-\hat{E}_1)+\beta_2(\hat{S}-\hat{E}_2)+\cdots+\beta_m(\hat{S}-\hat{E}_m)\\&+\beta_{n+1}(\hat{S}-\hat{E}_{n+1})+\beta_{n+2}(\hat{S}-\hat{E}_{n+2})+\cdots+\beta_k(\hat{S}-\hat{E}_k)\end{aligned} \tag{A.11}$$

Since

$$\sum_{i=0}^{m}\beta_i+\sum_{i=n+1}^{k}\beta_i=1 \tag{A.12}$$

It follows that

$$F=\hat{S}-(\beta_1\hat{E}_1\div\beta_2\hat{E}_2+\cdots+\beta_m\hat{E}_m+\beta_{n+1}\hat{E}_{n+1}+\cdots+\beta_k\hat{E}_k)$$

Since

$$\hat{S}=\sum_{i=1}^{n}\alpha_i\hat{E}_i=\sum_{i=1}^{m}\alpha_i\hat{E}_i+\sum_{i=m+1}^{n}\alpha_i\hat{E}_i \tag{A.13}$$

then

$$F=\sum_{i=1}^{m}\alpha_i\hat{E}_i+\sum_{i=m+1}^{n}\alpha_i\hat{E}_i-\sum_{i=1}^{m}\beta_i\hat{E}_i-\sum_{i=n+1}^{k}\beta_i\hat{E}_i$$

or

$$F=\sum_{i=1}^{m}(\alpha_i-\beta_i)\hat{E}_i+\sum_{i=m+1}^{n}\alpha_i\hat{E}_i-\sum_{i=n+1}^{k}\beta_i\hat{E}_i \qquad \text{(A.14)}$$

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